Large meridian instrument of the Samarkand Observatory and related measurements

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Abstract. The analysis of the astronomical data from the Zij-i Sultani demonstrates that the achievements of the Samarkand observatory represented the pinnacle of pre-telescopic astronomy. While the results are often presented with an excessive number of significant figures, these should be understood as the product of mathematical continuity rather than direct observational precision. By evaluating the physical limitations of the 40-meter meridian instrument and the inherent errors in determining the equinoxes, we can conclude that the systematic error of approximately 1′ was a remarkable feat for its time. Furthermore, this study clarifies that some values long attributed to Ulugh Beg in modern literature, such as the high-precision sidereal year, were actually derived from established constants like the precession rate of 1° per 70 years. Ultimately, the work of Ulugh Beg and his collaborators remains a testament to the rigorous application of geometric principles and historical data, where even the discrepancies with modern values fall within the rationally calculated margins of error for 15th-century science.

Keywords: Ulugh Beg, Samarkand observatory, Zij-i Sultani, meridian instrument, measurement accuracy.

Introduction

The Samarkand observatory, where Timur's grandson Ulugh Beg gathered many significant astronomers and mathematicians in the first half of the 15^{th} century, was an outstanding scientific project. Their observations and calculations resulted in the *Zij-i Sultani* (also known as the *Zij-i-Gurgani*), a collection of astronomical and trigonometric tables including a catalogue of 1018 stars. These tables were immensely influential over the following two centuries, including in Europe. The observatory was equipped with a variety of instruments, the most impressive of which was a meridian goniometric instrument with an arc radius of 40 meters. The history of the observatory, the contents of the *Zij-i Sultani*, attempts to reconstruct

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the architecture of the main building, and the details of the meridian instrument are the subject of extensive literature (see the brief bibliography at the end of the article; a more comprehensive list is provided by Matviyevskaya & Sokolovskaya, 1997).

In presenting the results of measurements and calculations, the *Zij-i Sultani* indicates angular values determined with the accuracy of arcseconds and time intervals calculated to fractions of a second. The purpose of this article is to evaluate the real capabilities of a large goniometric instrument, identify sources of error, and ultimately demonstrate that the results obtained at the Samarkand observatory were at the limit of pre-telescopic astronomy. In addition, I consider it important to clarify which results cited in modern literature were actually obtained by Ulugh Beg and his collaborators and which were calculated posthumously or in later periods.

Preliminary information

To make this text accessible to readers less versed in astronomical terminology, it is useful to provide it with a brief glossary of astronomical terms. The generally accepted system today is the heliocentric system, in which the Earth rotates on its axis and moves in an elliptical orbit around the Sun. All descriptions in ancient astronomy are made in the geocentric system, and I will refer to them as descriptions of *celestial phenomena*.

The tropical or solar year is the period of time during which the Sun returns to the same equinox, completing a full cycle of astronomical seasons. A year is roughly estimated to contain 365¼ days, but in reality, it is slightly less, and the refinement of this value was one of the most important problems of ancient astronomy. Claudius Ptolemy, in his *Almagest*, says that “finding the year’s time-length is the first of all the things demonstrated concerning the Sun” (III, 1), and all other astronomers echo him.

The sidereal or stellar year is the period of time during which the Earth makes one revolution around the Sun. In terms of celestial phenomena, it is the period of time during which the Sun, moving along the ecliptic, returns to the same point among the fixed stars. The sidereal year is approximately 20 minutes longer than the tropical year, which is due to the axial precession of the Earth; this difference is historically called *the precession of the equinoxes*. It was discovered by Hipparchus; due to its smallness, it is very difficult to measure this difference, and refining its value was a real challenge for ancient astronomers.

The true solar day is usually defined as the time interval from noon, when the Sun passes through the celestial meridian, until the following noon. The duration of the true solar day varies during the year for two reasons. First, the Earth moves around the Sun in an elliptical orbit, and this motion is uneven. Second, the Earth's axis is tilted relative to the plane of its orbit, and in celestial phenomena, the Sun moves relative to the fixed stars along the ecliptic, which is inclined to the celestial equator. Therefore, astronomers introduce the concept of *the mean Sun*, which is the name of the fictitious Sun that moves along the celestial equator at a constant rate, making one revolution between two vernal equinoxes in one tropical year. The mean solar day is the time interval between two noons defined by the mean Sun.

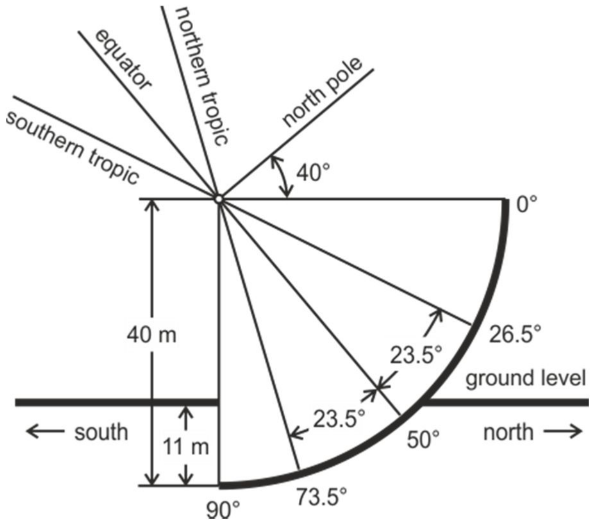
The sidereal or stellar day is the period of time during which the Earth makes one revolution around its axis; in celestial phenomena, this is the time during which the sphere of fixed stars makes one revolution.

The large meridian instrument of the Samarkand observatory

The most important astronomical instrument of the Samarkand observatory was the large meridian instrument intended for observations of the midday Sun. It had a measuring scale with a radius of 40.2 m. Only the underground part of the instrument, excavated by the Russian archaeologist Vassily Vyatkin in 1908, has survived to this day. The basic structure of the instrument, assuming it was a quadrant[1], is shown in the following figure. It shows the arc of its measuring scale, the lines of sight to both the tropics and the celestial equator, as well as the direction to the north celestial pole, perpendicular to the direction of the equator. The angles are presented approximately, with an accuracy of half a degree (see next page for a schematic representation).

Given the dimensions of the instrument, the value of one degree on the scale is 70.2 cm, one angular minute is equal to 11.7 mm, and the lost copper arc could have been marked with even smaller divisions. It is interesting to investigate how meaningful such precision in marking was. To do this, we will evaluate the measurement errors of the instrument. These errors are of two types: measurement errors and manufacturing inaccuracies.

[1] Most Russian-language texts on the Samarkand observatory are concerned with reconstructing its appearance: what the observatory building looked like and whether its main instrument was a quadrant or a sextant with an increased working arc. For the purposes of this paper, this distinction is insignificant, since the measurement methods in both cases were the same, and in the case of a quadrant, only the working part of the scale was subject to degree markings.



The standard measurement performed on such an instrument consists of establishing the angular distance between the Sun and the zenith at noon. To do this, an opaque screen with a circular aperture – a diopter – is installed at the top, in the center of the arc. The sunbeam, passing through this diopter, casts a sunspot onto the scale. This method was previously employed in the Fakhri sextant, a meridian astronomical instrument with a scale radius of 20 m, built in the 10th century by Abu Mahmud al-Khujandi in the city of Ray (near modern Tehran), and subsequently described by Abu Rayhan al-Biruni:

> Then Abu Mahmud Hamid ibn al-Khidr al-Khujandi built, by order of Fakhr ad-Dawla, two parallel walls along the meridian on Mount Tabarak in the outskirts of the city of Ray, with a distance of 7 cubits (~3.5 m) between them. Between the walls, he constructed an arch with a diopter at the top, the aperture of which had a diameter of a span (~20 cm). He placed the center of this diopter at the center of the sixth part of the circumference along the meridian between these two walls; the diameter of this arc was eighty cubits (~40 m). He lined the arc with wooden boards, then covered it with a brass surface and divided each degree of the complete circuit into 360 equal parts, each of which was equal to 10 seconds. The Sun penetrated through this diopter onto the meridian line. Abu Mahmud made a ring the size of the illuminated spot on the ground, marking its center with two intersecting diameters. He superimposed the circumference of the ring on that of the spot and, using the center of the ring, determined the distance between the Sun and the zenith. [al-Biruni 1966, p. 133; see also Bulgakov 1972, Oudet 1994]

It should be noted that the solar disk has a visible angular size of 30′; thus, even if the diopter of the Samarkand observatory had a pinhole aperture, the diameter of the sunspot on the scale would be 35 cm. However, a pinhole aperture has zero light-gathering power, so its size must be increased, which leads to an enlargement of the sunspot and penumbral blurring at its periphery. If the aperture of the diopter were 25 cm, then the external diameter of the spot would be 60 cm. It is unlikely that it was possible to determine the position of the center of such a spot with an error of less than 1 cm; this means that the accuracy of the reading on the scale was about 1′, and there was little utility in finer markings.

Regarding the manufacturing inaccuracies of the instrument itself, they were related to the fact that the diopter at a height of 40 m was not installed exactly at the center of the arc, and the arc itself deviated from an ideal circle. Al-Biruni wrote about the presence of such errors in the Fakhri sextant, and similar errors could not be avoided in the construction of the Samarkand observatory. In addition, there were errors related to the refraction of solar rays in the atmosphere. Thus, the presence of a systematic error of 1′ was the likely manufacturing accuracy for such an instrument. The following remark by Edward Kennedy is particularly significant here:

> The principle behind the exploitation of size was the fact that the bigger the distance between successive degrees, say, on a scale, the more the graduations that could be inserted, hence the higher the apparent precision. However, as the size increases, the difficulty of precise construction also increases, which tends to nullify the gain (Kennedy 1998, p. 283).

Zotti & Mozaffari (2020) proposed a reconstruction of the sighting device that moved along the arc of the large instrument of the Samarkand observatory. Their reconstruction was based on two schematic drawings from two manuscripts of al-Kashi's treatise *On Observational Instruments*. They state (p. 261) that

> both figures conspicuously show a sighting device, consisting of a long alidade with two pinnulae with tiny holes in their centres (the so-called "perpendicular pillar"), which is perpendicularly mounted on a wheeled "parallel implement" that can move up- and downwards on the upper surface of the sextant.

Let us consider whether such an auxiliary device could increase the accuracy of observations and allow for high-precision observations of not only the Sun but also, with the diopter removed, the Moon and the stars. There are two strong objections to this assumption. First, it would be difficult to maintain the position of such a wheeled cart on inclined rails, particularly at the top of the arc, where the weight of the alidade would pull the device downwards. Second, it would be extremely difficult to manufacture the brass rails of the arc so that the alidade, in any position of the cart, would be aligned with high accuracy toward the center of the arc. With even a slight deviation of the rails from the correct shape, the alidade would point above or below the center, and it would be very difficult to correct this error. Therefore, it appears that al-Kashi's drawing is largely speculative, and such a scheme would have little practical utility.

Basic measurements

To begin using the meridian instrument, it is necessary to establish *the geographic latitude* of the location where the observatory is situated. This latitude is equal to the angle at which the north celestial pole rises above the horizon, and it is also equal to the angular distance between the zenith and the point where the celestial equator intersects the celestial meridian. However, as the pole and the equator are theoretical constructs, and the corresponding angles must be determined through observing the Sun.

The idea behind such observations is based on the fact that the Sun passes through the celestial meridian at practically the same altitude for several days around the summer and winter solstices; thus, for approximately five days, this altitude change by less than 1½ arcminutes, which is within the limits of measurement accuracy. Therefore, the midday Sun altitude, measured on these days, gives the altitudes of the northern and southern tropics at their culminations. The altitude of the celestial equator and the geographic latitude of the observatory are equal to half the sum of these two altitudes, and *the obliquity of the ecliptic*, which is the angular distance from the celestial equator to each of the tropics, is equal to half the difference between these altitudes.

The geographic latitude of the observatory according to the *Zij-i Sultani* was 39°37′28″, whereas according to modern data, it is 39°40′29″. The error is 3′, which is consistent with the above estimate of measurement errors.

The obliquity of the ecliptic according to the *Zij-i Sultani* was 23°30′17″. Calculated using Bessel's formula for the year 1437, it was 23°30′49″. The error is 32″; the corresponding error in determining the difference in the angles of the Sun's culmination on the days of the winter and summer solstices was twice as large, amounting to about 1′.

The difference in the magnitudes of these two errors is explained by the fact that when calculating the sum of two measurements, the systematic error doubles, whereas when calculating their difference, it is reduced or even eliminated entirely.

Measuring the length of the tropical year

As stated above, the tropical year is the period of time from equinox to equinox. In a general sense, the equinox occurs when day and night are of equal length; in astronomy, it is defined precisely as the moment when the center of the solar disk crosses the celestial equator in its movement along the ecliptic. In order to measure the length of the tropical year, one must first be able to determine the moment of the equinox with the utmost accuracy.

Let us describe how this moment can be determined using a meridian instrument. For a certain number of days before and after the vernal equinox, daily observations of the altitude of the Sun above the horizon at the moment of its culmination should be conducted. As a result, a series of altitudes will be obtained, which can then be approximated by a linear function. By calculating the moment in time when the value of this function equals the height of the celestial equator, one can determine the moment of the equinox.

As stated above, all measurements of this kind were made with an accuracy of 1′, and an unavoidable systematic error might also be 1′. Since the Sun, at each subsequent noon near the vernal equinox, crosses the celestial equator at a point approximately 24′ higher than the day before, the accuracy of determining the moment of the equinox would be approximately one hour, but not higher.

Thus, if we determine two moments of the vernal equinox, separated by an interval of one year, the duration of the tropical year is found with an accuracy of two hours. In order to reduce this error to one minute, it is necessary to separate these two observations of the vernal equinox by an interval of 120 years, so that any astronomer making such a measurement will be forced to rely on the data of his predecessors. A further increase in accuracy requires a further extension of the time interval.

In this regard, two difficulties arise, which we must also learn to address. If two observations are separated by an interval of several hundred years, we must accurately calculate the number of days between them without losing or adding a single day; because the loss or gain of the day over an interval of 100 years would result in the determination of the length of the year with an accuracy of 1/100 of a day (approximately 15 minutes), and no better. To solve this problem, special calendar calculations are needed, possibly involving the coordination of different calendars from different eras; indeed, the entire first chapter of the introduction to the *Zij-i Sultani* is devoted to such calculations.

The second difficulty is that the observations of the equinoxes carried out by astronomers of the past usually took place at different observatories. The moment of the equinox was determined in each observatory according to its local time; in order to convert it from one observatory to another, it was necessary to know the difference in *the geographical longitude* of these locations. Determining the difference in longitude between two points was a very difficult problem for ancient geodesy, since it relied on the observation of the same lunar eclipse at both locations, with the determination of the beginning and end of the eclipse according to local time. Thus, in turn, required determining the time via the stars or by the position of the Moon.

Based on these considerations, it can be concluded that over an interval of 500 years – provided not a single day of this interval is lost or added, and assuming the beginning and the end of this interval, once reduced to a single time, are determined with an accuracy of one hour – the length of the year would be determined with an accuracy of 1/250 of an hour, which is approximately 15 seconds. Furthermore, no one could achieve a better result with these methods.

The problems that arise in such measurements were described by Claudius Ptolemy in his *Almagest* III.1:

> Thus, the extra amount can be perceived only when it is found added up over a longer period of time. And it must be divided among the intervening years of the interval, and it must be observed for a greater or smaller number of years than this same interval. The period of return will be obtained the more accurately the longer the time between the observations compared. This is the case not only with this period of return, but with all of them. For the error resulting from the weakness of the observations themselves, even if they are performed accurately, is small and very nearly the same as far as the senses are concerned, both for phenomena considered over a long time and for those considered over a short time. And this error of observation, when it is distributed over fewer years, makes the error in the length of the year greater, and similarly in multiples of it over a longer period of time; however, it makes the error in the length of the year smaller when distributed over a greater number of years. Therefore, it is properly thought sufficient if, when we consider how much the time between us and the old yet accurate observations can help in the approximation of the supposed periods of revolution, we try to introduce them with the others and do not willingly forego the proper verification. Finally, we may suppose that the establishing of dates for a whole long age, or for some great multiple of time between observations, is the work for another's love of wisdom and truth.

The tropical year in the *Zij-i Sultani*

The first passage concerning the length of the tropical year is found in the introduction to the *Zij-i Sultani*. I translate it here from the French edition by Sédillot (1853, p. 8):

> There are two types of civil days: first, the true day, which for the astronomers of our empire and of Western countries is counted from noon to the following noon. <…> The true civil day is one whose determination most concerns astronomers. Then comes the mean day, equal to the duration of a revolution of the eighth sphere, combined with the mean motion of the Sun, which, according to our observations, amounts to $0^\circ 59' 8'' 19''' 37'''' 43'''''$.

The eighth sphere, according to Ptolemy, is the sphere of the fixed stars; one of its revolutions constitutes a sidereal day. When its rotation is combined with the mean motion of the Sun, a mean solar day is formed. According to Ulugh Beg, if the duration of the tropical year is taken as 360°, a day is slightly less than 1°.

It is logical to determine from these data how many days there are in a tropical year. Ulugh Beg gives his data with an accuracy of five sexagesimal digits, which corresponds to nine decimal digits. In the calculations below, I provide eleven decimal digits for verification, writing the last two digits in a reduced font. Converting the sexagesimal notation to decimal and dividing 360 by the resulting number, we obtain a length of the tropical year of $365.242534 74^{\mathrm{d}}$. Multiplying this number by 86400, the length of the year in seconds is $31556955.002^{\mathrm{s}}$.

Examining the fractional part of this number, the hypothesis arises that seconds were, for Ulugh Beg, the smallest measure of time, and that the tropical year according to his data was $31556955^{\mathrm{s}} = 365^{\mathrm{d}}5^{\mathrm{h}}49^{\mathrm{m}}15^{\mathrm{s}}$, which is $10^{\mathrm{m}}45^{\mathrm{s}}$ less than 365¼ days. However, this assumption is inconsistent with the following statement from section IV.5:

> The true solar year, according to our observations, is calculated on average as $365^{\mathrm{d}}5^{\mathrm{h}}49^{\mathrm{m}}15^{\mathrm{s}}31'''48''''$ (Sédillot 1853, p. 214).

It is clear that the astronomers of the Samarkand observatory could not have measured the length of the solar year with such high precision. This result was likely derived from a calculation, where the last digits, exceeding any reasonable precision, were not discarded. It is not possible to reconstruct the initial data used for this calculation, as this would require knowing not only the years in which both equinoxes were observed but also the location where the first observation took place.

Another statement related to the length of the tropical year is found in section III.1:

> When we have any time expressed in true days, and we wish to convert it into mean days, we subtract, respectively, the mean longitude of the Sun and its true right ascension at the beginning of the given time from its mean longitude and its true right ascension at the end of the same time; we take the difference of the two remainders; we convert this difference into parts of mean hours in the proportion of $15°2'27''50'''49''''$ per hour, and we obtain in minutes of mean hours, the equation of time for the given interval. (Sédillot 1853, p. 133)

The sphere of fixed stars rotates by 15° in one hour. The difference of $2'27''50'''49''''$ arises due to the movement of the mean Sun relative to the fixed stars. Multiplying this difference by 24, we find that the mean Sun moves by $59'8''19'''36''''$ in one day, this represents the ratio of the length of a solar day to a tropical year of 360°. This is consistent with the results given in the two previous passages.

The sidereal year and the precession of the equinoxes in the *Zij-i Sultani*

The English-language Wikipedia article on *Ulugh Beg* contains the following passage, which has been reproduced on the internet in various languages at least sixty times:

> In 1437, Ulugh Beg determined the length of the sidereal year as $365.2570370...^{d}$ = $365^{d}6^{h}10^{m}8^{s}$ (an error of +58 seconds). In his measurements over the course of many years he used a 50 m high gnomon. This value was improved by 28 seconds in 1525 by Nicolaus Copernicus, who appealed to the estimation of Thabit ibn Qurra (826–901), which had an error of +2 seconds. However, Ulugh Beg later measured another more precise value of the tropical year as $365^{d}5^{h}49^{m}15^{s}$, which has an error of +25 seconds, making it more accurate than Copernicus's estimate which had an error of +30 seconds. Ulugh Beg also determined the Earth's axial tilt as 23°30′17″ in the sexagesimal system of degrees, minutes and seconds of arc, which in decimal notation converts to 23.5047°.

At the end of this passage there is a reference to pages 87 and 253 of Sédillot's translation (1853), but those pages discuss the obliquity of the ecliptic only. Consequently, I was unable to identify the source of the values mentioned above.

What did Ulugh Beg himself say about the length of the sidereal year and the precession of the equinoxes? The relevant fragment from the *Zij-i Sultani* (III.13) can be found in the book by Kary-Niyazov (1950, p. 279):

> In our catalogue, we refer the positions of the stars to the beginning of the year 841 AH [= 1437 CE], but one can find the position of each of them at any time, considering that they move forward by one degree every 70 solar years.

This text can also be found in the French translation by Sédillot (1853, p. 200), but instead of seventy (*soixante-dix*), it says sixty (*soixante*). This is clearly a misprint, as the Persian text published by Sédillot six years earlier (Sédillot 1847) reads "seventy years".

In Kary-Niyazov's book (p. 276), there is also the following statement:

> The length of the sidereal year was determined by Ulugh Beg as $365^{d}6^{h}10^{m}8^{s}$, whereas the actual length of this year according to Newcomb (1900) is $365^{d}6^{h}10^{m}6^{s}$. <...> Thus, in this instance, Ulugh Beg also achieved a high degree of accuracy.

This statement is accompanied by a reference to Bailly's treatise on the history of astronomy (Bailly 1787, p. 155), which is turn refers to his other work (Bailly 1785, p. 612):

> Ulugh Beg determined the motion of the fixed stars as one degree per 70 years, which was very close to the truth; from this, he established the annual motion at 51′26″ and the time of a complete revolution as 25200 years. <…> We also find the sidereal year to be $365^d6^h10^m8^s9'''23''''$, which is longer than ours by about 1^m10^s.

How did Bailly obtain the thirds and fourths in this result? I was unable to reconstruct them. Delambre (1819, Vol. 1), who followed this line of reasoning, wrote the following:

> The epoch of longitudes falls at the beginning of the year 841 AH, and the movement is one degree in 70 years (p. 207) <…> The accuracy of the solar tables proves that Ulugh Beg had conducted good observations. Greaves states that they were made using a great gnomon. In consequence of the movement that he gave to the Sun, his year was longer by 27^s, or $365^d5^h49^m15^s$ (p. 210).

Thus, the strange assertion that Ulugh Beg obtained his results using a 50-meter-high gnomon originates from John Greaves. (A gnomon is a type of sundial – a vertical stick or pillar that casts a shadow on a horizontal surface.) Of course, such a giant gnomon never existed, the device was, in fact, the large 40-meter meridian instrument.

The above-mentioned sidereal year of $365^d6^h10^m8^s$ can be obtained by a simple calculation based on the data given in the *Zij-i Sultani*. If the precession of the equinoxes is 1° per 70 years, then a full cycle of 360° lasts 25200 years. Dividing the number of seconds in an year by 25200, we find $1252^s = 20^m52^s$ for each year (it does not matter what duration of the year in seconds to take here, as only the first four significant digits affects this calculation). Adding 20^m52^s to the tropical year of $365^d5^h49^m15^s$, we obtain a sidereal year of $3365^d6^h10^m07^s$, which matches the result attributed to Ulugh Beg within an accuracy of one second.

General remark on measurement accuracy

In modern science it is common to present the results of measurements and calculations in the form of $A \pm \Delta$, with an explicit indication of the measurement error. The value of the error Δ usually contains one, sometimes two, significant figures; the result A should not contain excess significant figures that do not correspond to the actual error.

Ancient and medieval astronomers were not yet aware of such requirements for the presentation of their results, although they certainly had conceptions of how to increase measurement accuracy. Their results could contain superfluous significant figures, especially when these digits arose during intermediate calculations. If such an astronomical text contains the result of a time measurement presented with the precision of seconds, or an angular measurement presented with the precision of arcseconds, this does not mean that the actual measurements and calculations were performed with such accuracy; the redundant digits were simply not discarded.

Thus, when analyzing ancient astronomical texts, it is not enough to merely indicate the difference between a value measured or calculated by an ancient astronomer and the corresponding value according to modern data. It is also important to estimate the actual measurement error based on the capabilities of the instruments and the methods used. If the modern value falls within this error range, we can claim that the measurements were carried out correctly, within the limits of their accuracy. Furthermore, it may happen that the data obtained by an ancient astronomer differs from modern data by an amount significantly smaller than the margin of error of the measurements. In such cases, we should not necessarily marvel at the precision of the ancient measurements but rather acknowledge that such a small difference was obtained by chance – which, of course, does not detract from the work and achievements of our predecessors.

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