Cicero's inference from a non-axiomatic system

Miguel López-Astorga
University of Talca, Talca Campus (Chile)
milopez@utalca.cl

Abstract. Cicero posed a controversial claim. He proposed that the Stoics admitted an inference hard to accept. That inference is also incorrect in propositional calculus. It consists of two premises and a conclusion. The first premise is the negation of a conjunction. The second premise is the negation of one of the conjuncts of that conjunction. The conclusion provides that those premises allow deriving the truth of the other conjunct. My purpose here is not to analyze whether the inference can be valid in the general framework of Stoic logic. My intention is not to argue in favor of or against the idea that Cicero made a mistake either. The aim of the paper is only to show that a non-axiomatic logic can make inferences akin to that described by Cicero, and that, hence, an Artificial Intelligence system based on that logic can also do that. Two components of the logic are essential here: its inheritance copula and its concept of extensional difference.

Keywords: Cicero; extensional difference; inheritance copula; Non-Axiomatic Logic; Non-Axiomatic Reasoning System.

* Project ANID FONDECYT Regular Nº 1240010, "Modus Tollendo Tollens y condicionales de obligación: Un análisis de los efectos facilitadores del criterio estoico."

Introduction

Cicero thought that an inference with the following logical form was proposed within Stoic logic[1]:

(1) "Non et hoc et illud; non autem hoc, illud igitur"[2]:

Inference (1) provides that we cannot admit 'this' and 'that' at the same time. However, 'this' is not the case. Therefore, 'that' requires to be admitted.

[1] For an account, see, e.g., O'Toole & Jennings (2004).

[2] Cicero (*Topica*, XIV, 57).

DOI: 10.25205/1995-4328-2026-20-2-600-610

The problems regarding this structure are at least two. First, it does not seem to be related to the five 'indemonstrables' usually attributed to Stoic logic[3]. On the other hand, its logical form is not that of a correct inference in propositional calculus, as (2) is the case in the latter calculus[4].

(2) $\neg(p \land q) \land \neg p \nvdash q$

Where '¬' represents negation, '∧' denotes conjunction, and '⊬' indicates that the right formula does not logically derive from the left one.

Considering this, Cicero's inference in (1) has been interpreted in different ways. Some examples are very interesting. There are works that tried to show that the inference can make sense in the general framework of Stoic logic[5]. Even the theory of mental models was used as methodological support to argue a similar idea[6]. However, it was also said that the only important point regarding (1) is that it reveals that Cicero made a mistake because of his insomnia problems[7].

The aim of the present paper is not to address these debates. It is only intended to explain that (1) can work and be accepted within Non-Axiomatic Logic (NAL). NAL is a non-axiomatic logic in the sense none of its statements is true for sure. The truth values of the statements change as the system acquires knowledge[8]. This logic is used to build Non-Axiomatic Reasoning System (NARS). NARS is a computer system attempting to come to external cognitive responses akin to those of human beings[9]. Thus, if NAL can recreate inferences such as (1), NARS can capture those inferences, too.

So, I will not deal with Cicero's reasons for assuming (1). I will not analyze whether the inference makes sense within logical frameworks such as Stoic logic either. I will not even assess whether (1) is the result of a mistake. My goal will be to attempt to show that NAL and NARS can consider an inference that a human being (Cicero) assumed without difficulties, even if a mistake caused that. This way

[3] See, e.g., Diogenes Laertius (*Vitae Philosophorum*, 7, 79-81); also cited, e.g., in Boeri & Salles (2014, 216-217 & 228-229, Fragment 9.7).

[4] E.g., López-Astorga (2017).

[5] E.g., O'Toole & Jennings (2004).

[6] López-Astorga (2017); for the theory of mental models, see, e.g., Johnson-Laird (2023), Johnson-Laird, Byrne, & Khemlani (2024), and Johnson-Laird & Ragni (2025).

[7] As C. Normore stated; see also, e.g., O'Toole & Jennings (2004).

[8] E.g., Wang (2013).

[9] See also, e.g., Wang (2006).

to work is not new. There are already papers published addressing ideas coming from ancient philosophy by means of NAL and NARS[10].

I will do that with the two sections below. In the first section, I will review a little part of the machinery of NAL: the part necessary to address inferences such as (1). In the second section, I will explain how that minimal machinery allows making inferences with that formal structure.

A part of the machinery of NAL

A basic characteristic of NAL is that it is a term logic whose essential copula is the inheritance copula ('→')[11]. A statement in NAL typically has the structure of (3)[12].

(3) *Orange* → *Fruit* ⟨1, 0.9⟩

What (3) expresses is that *Orange* is an example of *Fruit*, and *Fruit* is a property of *Orange*[13]. The numbers provide to what extent we can accept the statement. Number 1 is the 'frequency', which in this case is the maximum frequency. It informs that all the oranges the system knows are fruits[14]. Number 0.9 refers to the 'confidence'. Considering how many oranges the system knows, it reveals the level to which the system can place confidence in (3). It is calculated taking the number of oranges reviewed and a constant into account[15]. 0.9 is a high level.

I have chosen 1 and 0.9 for, respectively, the frequency and the confidence of (3) because they are the numbers the system assigns to statements when they (the numbers) are not explicitly indicated. They are the numbers by default[16]. Besides, those numbers are useful to make the point of the present paper.

However, it is interesting to know the exact way frequency and confidence are estimated. Let us suppose that we have seen nine oranges, and that we have checked that those nine oranges are fruits. If F(3) is the frequency of (3), F(3) is calculated as follows[17]:

[10] E.g., López-Astorga (2025).
[11] E.g., Wang (2013).
[12] See also, e.g., Wang (2023).
[13] E.g., Wang (2013) or Wang (2023).
[14] E.g., Wang (2013) or Wang (2023).
[15] E.g., Wang (2013) or Wang (2023).
[16] E.g., Wang (2013).
[17] See Wang (2013, Definition 3.3).

$$F(3) = \frac{Number\ of\ oranges\ being\ fruits}{Number\ of\ oranges\ reviewed} = \frac{9}{9} = 1$$

On the other hand, if C(3) is the confidence of (3), it is calculated in this manner[18]:

$$C(3) = \frac{Number\ of\ oranges\ reviewed}{Number\ of\ oranges\ reviewed + 1} = \frac{9}{10} = 0.9$$

I add 1 to the denominator in C(3) because that denominator should consider a constant. I assign value 1 to that constant as it is the most habitual choice in NAL[19].

One more example can be useful to illustrate this to a greater extent. Let us suppose that we see 50 melons. 45 of those melons are green, 5 of them being a different color. The inheritance statement would be (4).

(4) *Melon* → *Green* ⟨0.9, 0.98⟩

The values of frequency and confidence for (4) are obtained by means of these divisions:

$$F(4) = \frac{Number\ of\ melons\ being\ green}{Number\ of\ melons\ reviewed} = \frac{45}{50} = 0.9$$

$$C(4) = \frac{Number\ of\ melons\ reviewed}{Number\ of\ melons\ reviewed + 1} = \frac{50}{51} = 0.98$$

NAL includes in its cumulative experience all the statements it knows so far. (3) and (4) would be two of the many statements NAL has in its cumulative experience in the current moment. As implied, the system can store more statements and change the values of frequency and confidence for the previous statements as time goes by. Likewise, NAL has a vocabulary containing all the terms it has learned. If the system knows (3) and (4), *Orange*, *Fruit*, *Melon*, and *Green* should be in its vocabulary[20].

---

[18] See Wang (2013, Definition 3.3).

[19] E.g., Wang (2013).

[20] For developments of these aspects of NAL, see, e.g., Wang (2011) or Wang (2013).

This logic also uses compound terms of different kinds. One of them is based on the concept of 'extensional intersection'. It allows linking terms, for example, adding a characteristic to a set of elements[21]. Extensional intersection enables to refer to sets such as that of the apples that are green. (5) expresses that set.

(5) $(\mathit{Apple} \cap \mathit{Green})$

The formula in (6) shows the meaning of (5). (6) resorts to First-Order Predicate Calculus. The latter calculus is not part of NAL. When that calculus is used in NAL, it always works as a metalanguage[22].

(6) $\forall \mathrm{x}\ \{[\mathrm{x} \rightarrow (\mathit{Apple} \cap \mathit{Green})] \Leftrightarrow [(\mathrm{x} \rightarrow \mathit{Apple}) \wedge (\mathrm{x} \rightarrow \mathit{Green})]\}$[23]

In (6), '$\forall$' is the universal quantifier and '$\Leftrightarrow$' is the biconditional symbol.

What (6) indicates is that $(\mathit{Apple} \cap \mathit{Green})$ is the set of all the elements that are examples of *Apple* and *Green* at the same time, that is, the intersection of the elements belonging to both *Apple* and *Green*.

However, the compound terms that are relevant for the present paper are others. They are the terms based on the concept of 'extensional difference'. Extensional difference is useful in the system to represent sets that do not have a characteristic[24]. For example, if I want to refer to the apples that are not red, I can express that as follows:

(7) $(\mathit{Apple} - \mathit{Red})$

Compound term (7) stands for the set of apples that are not red. Its meaning can also be shown using First-Order Predicate Calculus. That is made in (8).

(8) $\forall \mathrm{x}\ \{[\mathrm{x} \rightarrow (\mathit{Apple} - \mathit{Red})] \Leftrightarrow [(\mathrm{x} \rightarrow \mathit{Apple}) \wedge \neg(\mathrm{x} \rightarrow \mathit{Red})]\}$[25]

What (8) provides is that compound term $(\mathit{Apple} - \mathit{Red})$ is the set of all the elements that while they are examples of *Apple*, they are not examples of *Red*.

[21] E.g., Wang (2013).
[22] E.g., Wang (2013).
[23] See, e.g., Wang (2013, Definition 7.6).
[24] E.g., Wang (2013).
[25] See, e.g., Wang (2013, Definition 7.8).

Let us see some more examples.

(9) *Stoic* → (*Philosopher* ∩ *Greek*) ⟨0.8, 0.95⟩

Statement (9) indicates that the system has an identified proportion of Stoics that are philosophers and Greek at the same time with a frequency of 0.8 and a confidence of 0.95. These numbers could be obtained, for example, reviewing 20 Stoics and noting that 16 of them are Greek philosophers (F(9) = $\frac{16}{20}$ = 0.8; C(9) = $\frac{20}{21}$ = 0.95; if we keep considering the value of the constant to be 1).

On the other hand, in the case of (10),

(10) *Stoic* → (*Philosopher* - *Greek*) ⟨0.2, 0.95⟩

It indicates that the system has found a proportion of Stoics that are philosophers but not Greek with a frequency of 0.2 and a confidence of 0.95. These values apply to the same circumstance as that of (9). Now, instead of expressing that 16 of the 20 Stoics are Greek philosophers, (10) expresses that 4 of them while they are philosophers, they are not Greek (F(10) = $\frac{4}{20}$ = 0.2; C(10) = C(9) = $\frac{20}{21}$ = 0.95).

This is enough to explain how NAL, in the same way as Cicero, can come to the conclusion in (1) from its premises. The next section develops this point.

Cicero's inference and NAL

To address (1) from NAL, the first thing to do is to translate its premises into 'Narsese' language, that is, the language with which we can communicate with NARS via NAL. The first premise is (11).

(11) Non et hoc et illud.

Premise (11) gives several pieces of information. On the one hand, it provides that terms (12) and (13) are in the vocabulary of the system.

(12) *Hoc*

(13) *Illud*

Second, it claims that *Hoc* and *Illud* cannot happen at the same time. This means that while compound term (14) is not in the vocabulary of the system,

(14) $(Hoc \cap Illud)$

Compound terms (15) and (16) are also in the vocabulary of the system.

(15) $(Hoc - Illud)$

(16) $(Illud - Hoc)$

The non-existence of compound term (14) in the vocabulary of the system implies that *Hoc* and *Illud* do not share any examples. The existence of (15), which refers to the set of examples of *Hoc* that are not examples of *Illud*, and (16), which refers to the set of the examples of *Illud* that are not examples of *Hoc*, strengthens that.

The second premise is (17).

(17) Non autem hoc.

If we consider (12), (13), (15), and (16) to be the only terms in the vocabulary, the manner to express what (17) provides is (18).

(18) $T \rightarrow (Illud - Hoc)\ \langle 1, 0.9\rangle$

Statement (18) captures what (17) indicates because it informs that element *T*, which represents a particular element, is an example of (16). So, (18) establishes that *T* is an example of the compound term without examples of *Hoc*. This implies that (18) expresses a case in which *Hoc* cannot be allowed for. F(18) = 1 and C(18) = 0.9 because those values are the values by default.

Let us think about the conclusion now. It is (19).

(19) Illud igitur.

The question is whether everything described above allows coming to (19). The answer is positive. If (18) is correct, (20) is correct, too.

(20) $T \rightarrow Illud \langle 1, 0.9 \rangle$

Therefore, given what is indicated in (18), which corresponds to the second premise, (20) should be assumed. It is not hard to note that (20) entails that *Illud* occurs. Hence, it also entails that (19) should be admitted.

Against this, one might argue that (12), (13), (15), and (16) would not be likely to be the only terms in the vocabulary of the system. The second premise, that is, (17), could be expressed in Narsese in much more ways different from (18) (e.g., building compound terms based on extensional difference in which *Hoc* is the right or second simple term, and the left or first simple term is a term other than *Illud*). Many of those ways would not lead to (20).

I could respond to this objection. NAL and NARS can work with limited resources and information. The 'Assumption of Insufficient Knowledge and Resources'[26] is a basic assumption in NAL. So, it is not difficult to use the system considering only the data that can be derived from a part of its cumulative experience and the components of the system described in the previous section[27]. For example, it is possible to use the system considering only the data that can be derived from the premises in (1), that is, (11) and (17), and the components of the system described above. If only (11), (17), and the components of NAL explained in the present paper are taken into account, NAL comes to (19) via (20).

An example with thematic content can help show all this better. Let us think of inference (21).

(21) This man cannot be a slave and a Roman citizen at the same time
This man is not a slave

Therefore, this man is a Roman citizen

The first premise is (22).

(22) This man cannot be a slave and a Roman citizen at the same time.

It informs that (23), (24), (25), and (26) are in the vocabulary of the system.

(23) *Slave*

[26] E.g., Wang (2011).

[27] See also, e.g., Wang (2013).

(24) $(Roman \cap Citizen)$

(25) $[Slave \text{ - } (Roman \cap Citizen)]$

(26) $[(Roman \cap Citizen) - Slave]$

Likewise, (22) indicates that (27) is not included in the vocabulary of the system.

(27) $(Slave \cap Roman \cap Citizen)$

The second premise is (28).

(28) This man is not a slave.

If only (23) to (26) are considered, what (28) expresses can be provided by a statement such as (29).

(29) $T \rightarrow [(Roman \cap Citizen) - Slave]\ \langle 1, 0.9 \rangle$

Now, *T* represents a particular man. (29) could be more detailed, as NAL has a concrete manner to express specific individuals and proper names[28]. However, it is not necessary to explain that manner to achieve the aims of this paper. What is most important here is that if (29) is true, (30) is also true.

(30) $T \rightarrow (Roman \cap Citizen)\ \langle 1, 0.9 \rangle$

In this limited context, (30) makes the conclusion in (21), that is, (31), correct.

(31) This man is a Roman citizen.

Again, the objection could be that the context would be probably broader. Thus, (28) could be expressed in other ways. For example, it is possible to think that, although the man is not a slave, he can be a barbarian (and not a Roman citizen). But, as in the previous case, it can be claimed that both NAL and NARS were created to work with limitations. It would be easy to use the system considering only (22),

[28] E.g., Wang (2013, Definition 6.3).

(28), and the few components of NAL described above. In that circumstance, (31) would be the case in the system.

Conclusions

The inference Cicero attributed to Stoic logic can be made within NAL. The aim of this paper has not been to explore why Cicero admitted the inference and thought it was part of Stoic logic. My aim has not been to clarify whether he made a mistake either. The paper has not been even intended to review whether the inference makes sense in Stoic logic. My purpose has been only to show that we have logical resources to capture the inference, regardless of whether the inference is the result of a mistake or a valid schema in a certain logic.

Just a few components of NAL have been required to show that. One of them is its basic cupula, that is, the inheritance copula, which links examples to properties. One of the essential characteristics of NAL is that those links are always correct only to certain extent. Every statement in the latter logic has two truth values. One of them is frequency. The other one is confidence. Both are calculated by means of formulae. Beyond the fact that the system has a cumulative experience and a vocabulary, the other component needed to make the inference is the concept of extensional difference. That concept allows expressing that two terms have no shared examples.

Limiting its cumulative experience and using just those components, NAL can come to the conclusion in inferences with structures such as that of (1). This is important because it means that NARS, which is a computer system, can also do that: it can make inferences such as (1) as well. So, we can say that, at least for the case analyzed here, NARS can give the same responses as human beings, regardless of whether those responses are mistakes or not.

References

Boeri, M. D. and Salles, R. (2014) *Los filósofos estoicos. Ontología, lógica, física y ética*. Santiago de Chile.

Johnson-Laird, P. N. (2023) "Possibilities and human reasoning," *Possibilities Studies & Society* 1(1-2), 105-112.

Johnson-Laird, P. N., Byrne, R. M. J., and Khemlani, S. (2024) "Models of possibilities instead of logic as the basis of human reasoning," *Minds & Machines* 34(19), https://doi.org/10.1007/s11023-024-09662-4

Johnson-Laird, P. N. and Ragni, M. (2025) "Reasoning about possibilities: Modal logics, possible worlds, and mental models," *Psychonomic Bulletin & Review* 32(1), 52-79.

López-Astorga, M. (2017) "Cicero's mistake and denied conjunctions: Did Stoic logic include six *indemonstrables*?" *Schole* 11(2), 341-353.

López-Astorga, M. (2025) "The White Horse Paradox and Inheritance Logic," *Acta Baltica Historiae et Philosophiae Scientiarum* 13(1), 60-73.

O'Toole, R. R. and Jennings, R. E. (2004) "The Megarians and the Stoics," In: D. M. Gabbay and J. Woods, eds. *Handbook of the History of Logic, Volume 1. Greek, Indian and Arabic Logic*. Amsterdam, 397-522.

Wang, P. (2006) *Rigid Flexibility: The Logic of Intelligence*. Dordrecht.

Wang, P. (2011) "The assumptions on knowledge and resources in models of rationality," *International Journal of Machine Consciousness* 3(1), 193-218.

Wang, P. (2013) *Non-Axiomatic Logic: A Model of Intelligent Reasoning*. Singapore.

Wang, P. (2023) "The role of copulas in reasoning. Technical Report #17," *Temple AGI Team. Temple University*. https://cis.temple.edu/tagit/publications/TAGIT-TR-17.pdf