Modus Tollendo Tollens with obligation conditionals: Towards a deontic inheritance logic respecting the Stoic criterion

Miguel López-Astorga
Institute of Humanistic Studies, Research Center on Cognitive Sciences
University of Talca, Talca Campus (Chile)
milopez@utalca.cl

Abstract. We know that Modus Tollendo Tollens is a difficult rule to apply. We also know that there are circumstances in which people easily use it. One of those circumstances is whenever the conditional premise is an obligation conditional. On the other hand, the Stoic criterion of the conditional, that is, the proposal Chrysippus of Soli gave for the latter logical connective, has been related to Non-Axiomatic Logic and Inheritance Logic. My aim here is to try to show that obligation conditionals can be deemed as deontic inheritance statements in Non-Axiomatic Logic or Inheritance Logic. I will attempt to argue that it is possible to build a deontic inheritance logic with two essential characteristics. First, it respects the Stoic criterion of the conditional. Second, in consistence with the literature, it leads to the conclusion expected by classical logic when the conditional is an obligation.

Keywords: Inheritance Logic; Modus Tollendo Tollens; Non-Axiomatic Logic; obligation conditionals; Stoic criterion of the conditional.

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Introduction

Modus Tollendo Tollens is one of the logical schemata Chrysippus of Soli pinpointed[1]. It can be a cognitive problem: it is a correct rule in propositional logic, but it is not always applied[2]. It is usually expressed as the derivation in (1).

[1] E.g., Diogenes Laertius (*Vitae Philosophorum*, 7, 80).

[2] E.g., Byrne & Johnson-Laird (2009).

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(1) $\{P \Rightarrow Q, \neg Q\} \vdash \neg P$

In (1), '$\Rightarrow$' is the conditional in classical logic, '$\neg$' stands for negation, and '$\vdash$' represents logical deduction.

According to propositional calculus, (1) must be true regardless of its content. However, people do not often appear to think that. It seems that some contents lead individuals to the conclusion more easily than other contents. This is what happens with obligation conditionals, since they do not cause problems with Modus Tollendo Tollens. When the first premise expresses that if the first clause is true, it is mandatory for the second clause to occur, people tend to accept inferences such as (1)[3]. So, given an obligation conditional and the negation of its consequent, individuals habitually deem the negation of its antecedent as the conclusion[4].

This fact is a challenge for cognitive theories. It is not hard to explain why people reject (1). What is difficult is to explain why (1) is sometimes accepted (e.g., when the conditional is an obligation conditional). To explain why people rejects (1), it is enough to suppose that (1) is not a part of the schemata naturally working in human reasoning. This idea can be supported even assuming that the human mind follows a formal logic. For example, the mental logic theory[5] proposes that there is a mental logic leading the inferences human beings carry out. However, that mental logic does not include Modus Tollendo Tollens as one of its basic or 'core schemata'[6].

The problem is not this. The real problem is why there are occasions (such as those in which the conditional is an obligation conditional) in which individuals do make inferences such as (1). As far as I know, the accounts are few. One of the most relevant explanations at present is that of the theory of mental models[7]. According to this theory, regular conditionals effortless lead to a mental model in which the two clauses are the case. Any other mental model cannot be identified if there is no effort[8]. But the case of obligation conditionals is special[9].

Obligation conditionals allow quickly noting two mental models: that in which the two clauses are the case and that in which only the first clause is the case. By

[3] See also, e.g., Byrne (2005).

[4] See also, e.g., Cramer, Hölldobler, and Ragni (2021).

[5] E.g., Braine & O'Brien (1998).

[6] See also, e.g., O'Brien (2014); O'Brien (2021).

[7] E.g., Johnson-Laird (2023).

[8] See also, e.g., Johnson-Laird and Ragni (2019).

[9] Byrne (2005).

virtue of the latter model, individuals identify the forbidden circumstances (those matching it)[10]. Although the proponents of the theory of mental models refuse that models can be expressed by means of logical formulae (mental models represent reality, and they are not formulae[11]), to simplify, I will describe the thesis of the theory of mental models saying that a regular conditional such as (2).

(2) $P \Rightarrow Q$

Easily enables to note mental model (3).

(3) $\Diamond(P \wedge Q)$

Symbol '$\Diamond$' in (3) expresses possibility, but the theory of mental models does not understand 'possibility' exactly as in modal logic[12]. On the other hand, '$\wedge$' works as conjunction.

However, following, for example, Byrne[13], an obligation conditional such as (4).

(4) $O(P \Rightarrow Q)$

Where 'O' indicates obligation.

Leads without difficulties to (5).

(5) $Per(P \wedge Q) \wedge \neg Per(P \wedge \neg Q)$

Where 'Per' stands for permission.

I will not argue against this account. I will only propose another alternative based on Inheritance Logic and Non-Axiomatic Logic[14]. My aim is to show that formulae such as (4) can be understood from Inheritance Logic (IL) and Non-Axiomatic Logic (NAL). That will allow me to present a Deontic Inheritance Logic (DIL). As described below, DIL can account for why Modus Tollendo Tollens is hard when

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[10] Byrne (2005).

[11] See also, e.g., Johnson-Laird (2010); Johnson-Laird (2012).

[12] For the exact meaning of 'possibility' in the theory of mental models, see, e.g., Johnson-Laird (2023); Johnson-Laird and Ragni (2019).

[13] Byrne (2005).

[14] E.g., Wang (2013).

the first premise is such as (2), and why it is not difficult when that very premise is such as (4).

In addition, I will support the idea that DIL is coherent with the view Stoic logic has about the conditional[15]. So, my thesis will be that DIL is not only consistent with what the cognitive science literature provides about Modus Tollendo Tollens, but also with logical proposals from the past such as Stoic logic.

The sections of this paper will be the following. First, I will describe IL. Then, I will explain how NAL derives from IL. Third, I will show the way Stoic logic has been related to NAL. Fourth, I will introduce DIL. In the last section, I will argue that DIL can account for what happens with Modus Tollendo Tollens when its conditional is an obligation conditional. I will also give reasons for accepting relations between Chrysippus'view of conditionals and DIL.

Idealized inheritance statements in IL

IL is an idealization. It is an idealization of NAL. Nevertheless, perhaps to understand NAL, the best option is, as Wang does[16], to consider IL first. IL consists of inheritance statements such as (6).

(6) "$S \rightarrow P$"[17]

In (6), 'S' refers to the subject of the statement, '$\rightarrow$' denotes the copula providing the inheritance relation, and 'P' stands for the predicate of the statement.

Inheritance statements such as (6) are based on concepts such as 'extension' and 'intension'. But those concepts are not interpreted in IL as usual in the logical literature. Let 'S^E', 'P^E', 'P^I', and 'S^I' be, respectively, the extension of the subject, the extension of the predicate, the intension of the predicate, and the intension of the subject. (7) holds in IL.

(7) "$(S \rightarrow P) \Leftrightarrow (S^E \subseteq P^E) \Leftrightarrow (P^I \subseteq S^I)$"[18]

In (7), '$\Leftrightarrow$' is the symbol for biconditionality in logical calculus.

For example, let us suppose (8), (9), (10), and (11).

[15] E.g., Mueller (1978).

[16] Wang (2013).

[17] Wang (2013,14, Definition 2.2).

[18] Wang (2013, 20, Theorem 2.4).

(8) $Rice \rightarrow Cereal$

(9) $Corn \rightarrow Cereal$

(10) $Wheat \rightarrow Cereal$

(11) $Cereal \rightarrow Vegetable$

In IL, this means that (12), (13), (14), and (15) hold.

(12) $\{Rice, Corn, Wheat\} \subseteq \mathrm{C}^{\mathrm{E}}$

Where 'C^{E}' represents the extension of 'Cereal'.

(13) $\{Cereal\} \in \{\mathrm{R}^{\mathrm{I}} \cap \mathrm{CO}^{\mathrm{I}} \cap \mathrm{W}^{\mathrm{I}}\}$

Where 'R^{I}', 'CO^{I}', and 'W^{I}' are, respectively, the intensions of 'Rice', 'Corn', and 'Wheat'.

(14) $\{Cereal\} \in \mathrm{V}^{\mathrm{E}}$

Where 'V^{E}' stands for the extension of 'Vegetable'.

(15) $\{Vegetable\} \in \mathrm{C}^{\mathrm{I}}$

Where 'C^{I}' indicates the intension of 'Cereal'.

An important set in IL is the set of what the system knows, that is, the set of inheritance statements the system knows. That set is called '*K*'[19]. Following the previous examples, (16) is correct in IL.

(16) $\{(8), (9), (10), (11)\} \subseteq K$

Besides, inheritance statements are transitive[20]. By virtue of transitivity, it is possible to derive new inheritance statements from the statements that are already in *K*. From (8), (9), (10), and (11), we can deduce (17), (18), and (19).

[19] Wang (2013, Definition 2.4).
[20] Wang (2013, Definition 2.2).

(17) *Rice* → *Vegetable*

(18) *Corn* → *Vegetable*

(19) *Wheat* → *Vegetable*

Lastly, IL accepts 'Closed-World Assumption'[21]. This assumption is based on Russell and Norvig's idea that 'unknown' should be deemed as 'false'[22]. Therefore, it means that what is not in *K* (or it cannot be deduced from *K*) is not true. Thus, the system can respond to questions such as "S → P?", "S → ?", and "? → P". But when there is no answer to them, that is, when, respectively, 'S → P' is not in *K* or it cannot be deduced from *K*, no statement with 'S' as its subject is in *K*, or no statement with 'P' as its predicate is in *K*, the system returns 'NO'[23].

Realistic inheritance statements in NAL

IL might not be useful to work as the human mind in some circumstances. It can have difficulties in situations in which the resources or knowledge do not suffice. This is the reason why NAL is derived from IL in works authored by Wang[24]. There are relevant differences between IL and NAL. While IL follows Closed-World Assumption, NAL admits the 'Assumption of Insufficient Knowledge and Resources' (AIKR)[25].

NAL can describe situations in which people progressively acquire pieces of evidence without getting total knowledge. Let us suppose that we have seen ten mammals: a lion, a dog, a cat, a rabbit, a rhinoceros, a human being, a gorilla, a bear, a whale, and a bat. Although most of them are terrestrial, that information does not enable us to state (20).

(20) *Mammal* → *Terrestrial*

The whale and the bat are the animals causing problems: whales live in the sea and bats can fly.

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[21] Wang (2013).

[22] Russell and Norvig (2010).

[23] Wang (2013, 23, Definition 2.10).

[24] E.g., Wang (2013); Wang (2023).

[25] See also, e.g., Wang (2011).

If we move from IL to NAL, we can capture cases such as this one. In NAL, inheritance statements such as (6), that is, ‘S → P’, are transformed into (21).

(21) “$S \rightarrow P \langle f, c\rangle$”[26]

What (21) adds to (6) is a truth-value. In IL inheritance statements are either true, with truth-value 1, or false, with truth-value 0 (as said, it depends on whether they are in K). In NAL there are values for frequency (f) and confidence (c)[27]. We have formulae to calculate both.

Let ‘w’, ‘w^+’, and ‘k’ be, respectively, the total evidence the system knows, the positive evidence the system knows, and a constant whose value can be, for example, 1[28]. The formulae would be “$f = w^+/w$” and “$c = w/(w + k)$”[29].

Considering the previous example, it is easy to calculate w and w^+. There is a formula for that too: “$w^+ = |S^E \cap P^E| + |P^I \cap S^I|$”[30]. Since we have seen ten animals, $w = 10$. Taking the first summand of the latter formula, ‘$|S^E \cap P^E|$’, into account, we can note that $w^+ = 8$. This is because only eight of the ten animals we have checked are both mammals and terrestrial. The second summand, ′$|P^I \cap S^I|$’, could refer to additional evidence. For example, if K included the information that both mammals and terrestrial beings are living organisms, that would be also positive evidence to consider. But to make the point of this paper, it is enough to pay attention to the first summand. Based on that set, the values of f and c for (20) would be those in (22).

(22) *Mammal* → *Terrestrial* ⟨0.8, 0.9⟩

This is because $f = 8/10 = 0.8$, and $c = 10/(10 + 1) = 10/11 = 0.9$.

NAL foresees that the system can increase its knowledge. For that reason, the values of f and c can always change. NAL has rules to update those values. It also has rules to make inferences such as deductions, inductions, abductions, etc[31]. But as far as the aims of this paper are concerned, just two more characteristics of NAL require to be mentioned. One of them is that NAL is not only one logic. There are

[26] Wang (2013, 40, Definition 3.8).
[27] Wang (2013, Definition 3.3).
[28] See, e.g., Wang (2013) for reasons for giving that value to k.
[29] Wang (2013, 29, Definition 3.3).
[30] Wang (2013, 28, Definition 3.2).
[31] See, e.g., Wang (2013).

different layers of NAL with different rules and grammars[32]. The components described above correspond to the first layer, and they suffice for my goals here. On the other hand, whenever a statement does not have its value of frequency or its value of confidence, the system assigns default values: $f = 1.0$ and $c = 0.9$[33].

Chrysippus of Soli and NAL

The Stoic criterion for the conditional, that is, the requirement Chrysippus of Soli proposed for the latter connective[34] has been related to NAL[35]. This has been done because that relation allows dealing with the Stoic conditional from a computer program. NAL is a logical system leading to the building of Non-Axiomatic Reasoning System (NARS). NARS is a computer program trying to come to conclusions in a context akin to that of human beings, that is, a context characterized by AIKR[36].

To relate the Stoic view of the conditional to NAL, two suppositions are necessary. The first one is that Chrysippus not only supported the 'connexivist view' of the conditional[37] but also the 'inclusive view'[38]. The second one is to accept the difference between strong and weak conditionals in Stoic logic that Sedley[39] indicates[40].

The connexivist and the inclusion views are two of the four interpretations of the conditional Sextus Empiricus described[41]. The connexivist view[42] is usually attributed to Chrysippus of Soli[43]. It provides that the negation of the consequent, or second clause, should be incompatible with the antecedent, or first clause. On the other hand, the inclusion view proposes that the consequent, or second clause, should be contained within the antecedent, of first clause. The assumption consists of assuming that Chrysippus argued in favor of the two criteria, and that both were the same. It can be thought that if the consequent, or second clause, is included in

[32] For a description of those layers, see, e.g., Wang (2013).

[33] E.g., Wang(2013).

[34] E.g., O'Toole and Jennings (2004).

[35] López-Astorga (2024).

[36] See also, e.g., Wang (2006).

[37] E.g., O'Toole and Jennings (2004).

[38] This view is described in Sextus Empiricus (*Pyrrhoniae Hypotyposes*, 2, 112); see O'Toole and Jennings (2004).

[39] Sedley (1984).

[40] These are the two assumptions in López-Astorga (2024) to relate Stoic logic to NAL. My explanation below follows that in López-Astorga (2024).

[41] O'Toole and Jennings (2004).

[42] Which is in Sextus Empiricus (*Pyrrhoniae Hypotyposes*, 2, 111).

[43] See also Gould (1970).

the antecedent, or first clause, that implies that the negation of the former cannot be consistent with the latter. The acceptance of this assumption is based on works such as those of Kneale and Kneale[44], Lenzen[45], or Long and Sedley[46][47]. Thus, given that in NAL inheritance statements represent a relation in which the intension of the predicate is contained within the intension of the subject, we can say that the Stoic conditional is not different from inheritance statements such as (6) or (21)[48].

As far as Sedley's account about Stoic weak conditionals and Stoic strong conditionals[49] is concerned, the point is that both cannot be expressed in the same way. If the conditional relation between P and Q is not clear, that is, it is not obvious that ¬Q causes ¬P to happen (or that the meaning of P includes that of Q), the relation must be expressed as 'it is not the case that P and not Q'. However, if the relation is clear, that is, it is evident that ¬Q and P are incompatible (and that the meaning of P includes that of Q), the relation must be expressed as 'if P then Q'. This difference can be understood from NAL as a difference between frequency values. The Stoic strong conditionals are those for which $f = 1$, that is, for which $w^+ = w$. The Stoic weak conditionals are those for which $f < 1$, that is, for which $w^+ < w$[50].

How all of this can be computationally treated has been described. By means of a function written in Common Lisp language, a program can choose the appropriate way to express a Stoic conditional relation. The function only needs three data to be indicated: the first and second clauses and the value of frequency[51]. That function is reproduced in Appendix 1. In that appendix, I also develop the function from my arguments in the present paper.

This shows how Chrysippus' criterion can be addressed from NAL and NARS. In the next section, I will present DIL. My purpose is that DIL is consistent with both the literature about Modus Tollendo Tollens with obligation conditionals and this account about the Stoic requirement for the conditional.

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[44] Kneale and Kneale (1962).

[45] Lenzen (2019).

[46] Long and Sedley (1987).

[47] In fact, works such as those are the works López-Astorga (2024) cites to support the idea that, in Chrysippus' view, the connexivist and the inclusion requirements are the same.

[48] López-Astorga (2024).

[49] Sedley (1984).

[50] López-Astorga (2024).

[51] López-Astorga (2024).

A deontic inheritance logic

To insert obligation conditionals such as (4) into IL and NAL, it is needed to consider how those conditionals can work as inheritance statements. Since in statements such as (6) or (21), *S* and *P* are terms, and not sets[52], they can refer to actions or circumstances. For example, let us think about a sentence such as (23).

> (23) "If a person is drinking beer, then the person must be over 19 years of age"[53].

This is an obligation conditional that often gives good results in tasks with the structure of Modus Tollendo Tollens. It can be deemed as an inheritance statement such as (24).

> (24) *Beer* → *Over*

In (24), 'Beer' means 'to drink beer' and 'Over' represents 'to be over 19 years of age'.

To be an inheritance relation, (24) requires (25) and (26) to hold.

> (25) $B^E \subseteq O^E$

In (25), 'B^E' stads for the extension of 'Beer' and 'O^E' refers to the extension of 'Over'.

> (26) $O^I \subseteq B^I$

In (26), O^I indicates the intesion of 'Over' and 'B^I' is the intension of 'Beer'.

This is not a problem. If people under 19 years old are prohibited from drinking alcohol, (25) and (26) should be the case. As far as *f* and *c* are concerned, we must note that obligations are not necessarily built from experience. One might think that a statement such as (24) does not arise from people's observation: it does not depend on the number of people over 19 years old that have been reviewed to check whether they drink beer. This is because it does not depend on the elements of B^E, B^I, O^E, and O^I in the past. When an obligation is provided, it is expected that the obligation is followed, whether it has been followed in the past or not. In fact, in standard deontic logic, propositions such as (27) are not usually admitted[54].

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[52] Wang (2013).

[53] Cramer et al. (2021, 2337); coming from Griggs and Cox (1982).

[54] E.g., Forrester (1996, Proposition SDLX4).

(27) OP $\Rightarrow$ P

In standard deontic logic, it is not generally accepted that the fact that something is mandatory implies the fact that it is the case[55]. The validity of an obligation is independent of its compliance. Therefore, obligations do not necessarily arise from experience or are truer if people fulfill them.

As mentioned, if a statement such as (24) does not have a truth-value, NAL should assign $f = 1$ and $c = 0.9$ to it. Those values are already high. However, in the case of an obligation, f is not relevant, since positive evidence is not relevant. On the other hand, c should not be addressed either, as the acceptance of an obligation does not depend on the amount of evidence about it we have.

From this point of view, given that IL is linked to ideal situations, to deem obligation conditionals as inheritance statements in IL coexisting with inheritance statements in NAL seems the most appropriate option. From now on, I will use 'DIL' to refer to this combination of obligation conditionals as inheritance statements in IL with inheritance statements in NAL.

An important point is that, despite what was said, we can attribute values for f and c to inheritance statements corresponding to obligations in IL. The latter idea is not hard to argue. This is because "…if a statement has truth-value *true* in IL, then it has truth value $\langle 1, 1\rangle$ in NAL"[56]. In IL, evidence is complete. When evidence is complete, "$w \to \infty$" and "$c = 1$"[57]. In addition, given that IL assumes Closed-World Assumption, every statement in IL is undoubtedly true, that is, $f = 1$ for them. We can say that $w^+ = w$ in IL. So, $w^+/w = 1$ in IL[58]. Hence, we can also claim that the inheritance statements in IL referring to obligation conditionals and coexisting with other inheritance statements in NAL have a structure such as that of (28).

(28) $S \to P \langle 1.0, 1.0\rangle$

But perhaps a better way to express (28) is (29).

(29) $S \to_{\mathrm{obl}} P$

[55] Forrester (1996).

[56] Wang (2013, 129, italics in text).

[57] Wang (2013, 34).

[58] This identity between w and $w+$ is also considered in rules of NAL such as the 'exemplification rule'; see, e.g., Wang (2013, 59-62).

In (29), '$\rightarrow_{obl}$' indicates that the statement is an obligation, and that reasoning is deontic. '$\rightarrow_{obl}$' keeps expressing an inheritance relation. However, that relation is deontic. In it, $f = 1$ and $c = 1$, and those values are never going to change by virtue of future evidence. This is because with '$\rightarrow_{obl}$' the system reasons about what should be, that is, about ideal situations, not about what is really. Because of AIKR, the values of f and c can always change in NAL. That is the reason why statements such as (29) should be dealt with as statements just under the rules and grammar of IL, even though they coexist with statements under the rules and grammar of NAL.

DIL can explain the results reported in the literature on obligation conditionals in tasks of Modus Tollendo Tollens. The next section develops this point.

DIL, obligation conditionals, and Modus Tollendo Tollens

DIL deems obligation conditionals as inheritance statements such as (29), that is, ultimately, as inheritance statements in IL. A task of Modus Tollendo Tollens with this kind of statements can be represented as (30).

(30) $S \rightarrow_{obl} P$
$\quad ? \rightarrow_{obl} \sim P$

In (30), '$\sim P$' tries to denote the term contrary to P. What I mean by this is that the relation between P and $\sim P$ should be that in (31).

(31) $\{P^E \cap \sim P^E\} = ø$

Where '$\sim P^E$' stands for the extension of $\sim P$[59].

If (29) and (31) are the case, then '$S^E \subseteq \sim P^E$' is not the case. Therefore, (32) is not possible.

(32) $S \rightarrow_{obl} \sim P$

Thus, IL can respond to the question in (30) with any term 'x' that the system knows, provided that (33) holds.

(33) $S \neq x$

[59] For a discussion about what negations are in IL and NAL, see, e.g., Wang (2013, Chapter 9).

Furthermore, if the question is (34),

(34) $S \rightarrow_{\text{obl}} {\sim}P$?

The answer must be 'NO'.

These responses the system must give are compatible with inferences with the structure of Modus Tollendo Tollens. They express that, whenever the term contrary to the predicate of an inheritance statement is considered, the initial subject of the statement cannot be the case.

On the other hand, this continues to be coherent with the Stoic criterion. If obligation conditionals in DIL are as statement (28), or, if preferred, (29), that means that they are statements with f = 1, that is, fulfilling Chrysippus' criterion. So, in Stoic logic, they should be expressed as conditionals, and not as negated conjunctions. But if they are expressed as conditionals, the rule of Modus Tollendo Tollens can be applied to them in the Stoic system.

A Stoic weak conditional, which is expressed as a negated conjunction, only allows applying another schema: Modus Ponendo Tollens I[60]. As it is known, the structure of Modus Ponendo Tollens I is that in (35).

(35) $\{\neg(P \wedge Q), P\} \vdash \neg Q$

In propositional calculus, this does not make any points. (36) and (37) hold in the latter calculus.

(36) $P \Rightarrow Q \vdash \neg(P \wedge \neg Q)$

(37) $\neg(P \wedge \neg Q) \vdash P \Rightarrow Q$

But (36) and (37) do not appear to be the case in Stoic logic. If it is necessary to distinguish between strong and weak conditionals, that means that they are sentences with different senses. Modus Tollendo Tollens can be used only with a strong conditional in Stoic logic. If the conditional is weak, the rule to apply is Modus Ponendo Tollens I. My account of DIL respects that because it is about statements in which f = 1, that is, statements corresponding to Stoic strong conditionals.

[60] See Diogenes Laertius (*Vitae Philosophorum*, 7, 80).

One might think that both Modus Tollendo Tollens and Modus Ponendo Tollens I can lead to the same conclusion if the second premise is the negation of the second clause. (38) is a version of (35).

(38) $\{\neg(P \wedge \neg Q), \neg Q\} \vdash \neg P$

However, we cannot forget that Stoic weak conditionals correspond to inheritance statements in which $f < 1$. Hence, although NAL also has rules to respond to questions such as '? → P', the answers to those questions cannot be taken as completely correct answers. If $f < 1$, then $w^+ < w$. That means that not all the pieces of evidence support the statement. On the contrary, in DIL Modus Tollendo Tollens is applied because $f = 1$, and the answers are correct with no doubts. So, DIL appears to be consistent with essential requirements of Stoic logic.

As mentioned, in the literature, a very simple code in Common Lisp to deal with the Stoic conditional from NAL has been presented[61]. That code enables to express a sentence as a conditional or a negated conjunction by virtue of its frequency value. In Appendix 1, I will present a new function to complement that code. I will develop the code to also allow applying Modus Tollendo Tollens when the conditional is strong and includes 'if' and 'then'. But the main conclusion of all this seems to be obvious: the fact that Modus Tollendo Tollens is acceptable in DIL because in DIL obligations have $f = 1$ implies that DIL have two important characteristics. First, it is compatible with the literature about Modus Tollendo Tollens and obligation conditionals. Second, it respects Chrysippus' criterion as well.

Conclusions

A fact seems to be proved in the literature: while the inferences with Modus Tollendo Tollens are usually hard to accept, that changes when the conditional is an obligation. I have tried to address this issue from NAL in this paper.

NAL is based on IL and linked to a computer program such as NARS. If we understand conditionals as inheritance statements, and in particular, obligation conditionals as inheritance statements with $f = c = 1$, the explanation is not difficult. The contrary to the predicate in the latter inheritance statements implies that a term is the case: a term whose extension does not include the initial subject or the extension of the initial subject. Therefore, if the opposite of the predicate is the case, its subject cannot be the initial subject. Otherwise, the inheritance statement would be false.

[61] López-Astorga (2024).

Another important point is that, as also affirmed in the literature, the inheritance statements in which *f* = 1 are statements following Chrysippus' requirement. This means that the system I have called 'DIL' is coherent with both the results in the literature on Modus Tollendo Tollens and obligation conditionals, and the Stoic criterion for conditional sentences.

Further development of DIL remains pending. A priori and after the above, it seems that DIL could work well with NAL. However, the review of the inferential processes a logic such as DIL could make and how DIL could be really understood from NAL is still pending.

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APPENDIX 1

A very easy function in Common Lisp has been presented in the literature[62]. It allows expressing conditionals according to what, following Sedley[63], was the Stoic opinion. The function is:

```
"(DEFUN CHRYSIPPUS (L1 L2 N)
        (IF (= N 1) (APPEND '(IF) L1 '(THEN) L2)
                (APPEND '(IT IS NOT THE CASE THAT) L1 '(AND NOT) L2)))"
```
[64].

In the latter function, 'L1' is the first clause or subject, 'L2' represents the second clause or predicate, and N = *f*.

If we write,

CHRYSIPPUS '(THIS PHILOSOPHER IS A PYTHAGOREAN) '(THIS PHILOSOPHER IS A MATHEMATICIAN) 1

The system responds,

(IF THIS PHILOSOPHER IS A PYTHAGOREAN THEN THIS PHILOSOPHER IS A MATHEMATICIAN)

[62] López-Astorga (2024).

[63] Sedley (1984).

[64] López-Astorga (2024, 55).

However, if we write,

CHRYSIPPUS '(I AM EATING RICE) '(CARROT) 0.9

The answer is,

(IT IS NOT THE CASE THAT I AM EATING RICE AND NOT CARROT)

This code can be improved enabling the use of Modus Tollendo Tollens when the conditional is strong. My proposal is (I keep using Common Lisp, LispWork Personal Edition),

```
(DEFUN CHRYSIPPUSMTT (L1 L2 L3 N)
        (IF (AND (EQUAL (CAR (CHRYSIPPUS L1 L2 N)) 'IF)
                (EQUAL (CAR L3) 'NOT)
                (EQUAL (CDR L3) L2))
            (CONS 'NOT L1)
           '(REJECTED OR NOT ACCEPTED)))
```

L1 and L2 continue to be the first and second clauses (i.e., the subject and the predicate). N also keeps being *f*. L3 is the second premise.

Function CHRYSIPPUSMTT negates the first clause or subject if three conditions are fulfilled. The conditions are these:

1. L1, L2, and N should allow building a strong conditional using function CHRYSIPPUS.
2. L3 must starts with 'NOT'.
3. L3 without 'NOT' must be identical to L2.

If at least one of these conditions is not the case, the system returns '(REJECTED OR NOT ACCEPTED)'.

If we write,

CHRYSIPPUSMTT '(PYTHAGOREAN) '(MATHEMATICIAN) '(NOT MATHEMATICIAN) 1

The system gives,

(NOT PYTHAGOREAN)

Let us suppose a fictional context in which most Pythagorean philosophers are mathematicians, but not all of them. Thinking about that context, we can write,

CHRYSIPPUSMTT '(PYTHAGOREAN) '(MATHEMATICIAN) '(NOT MATHEMATICIAN) 0.9

The response would be,

(REJECTED OR NOT ACCEPTED)

The answer will be the same if we commit the affirming the consequent fallacy and write,

CHRYSIPPUSMTT '(PYTHAGOREAN) '(MATHEMATICIAN) '(MATHEMATICIAN) 1

Finally, if the content of L3 without 'NOT' does not match that of L2, the response will continue to be the same. That will happen if we write,

CHRYSIPPUSMTT '(PYTHAGOREAN) '(MATHEMATICIAN) '(NOT LOGICIAN) 1

Of course, if more than one condition is not met, that is, if two conditions are not met, or the three conditions are not met, the answer will be '(REJECTED OR NOT ACCEPTED)' as well.

Functions CHRYSIPPUS and CHRYSIPPUSMTT are both very simple. My only goal with this appendix is to show that what has been proposed in this paper does not make the computational treatment of Chrysippus criterion more difficult. The literature has already linked the cases of $f = 1$ in NAL to the stoic requirement for the conditional. To relate the cases of $f = 1$ in NAL to the acceptance of Modus Tollendo Tollens (as this paper does) is not a problem for the link in the literature.