# CICERO'S MISTAKE AND DENIED CONJUNCTIONS: DID STOIC LOGIC INCLUDE SIX *INDEMONSTRABLES?*

MIGUEL LÓPEZ-ASTORGA
Institute of Humanistic Studies "Juan Ignacio Molina"
University of Talca, Chile
milopez@utalca.cl

ABSTRACT. Cicero attributes an inference schema that is not logically valid to Stoic logic. This fact has been deemed as a Cicero's mistake. However, in this paper, I assume the thesis that Cicero was not actually wrong and that, by means of his schema, he could be showing us interesting aspects of Stoicism that are not clear in the ancient sources. In this way, I use the mental models theory as a methodological tool and argue that the case of Cicero's mistake allows us to suppose that there were six *indemonstrables* in Stoic logic, and not five.

KEYWORDS: Cicero; denied conjunction; *indemonstrable*; mental models; Stoic logic.

## Introduction

In *Topica* 13.57, Cicero states that Stoic logic includes an inference schema in which the first premise is a denied conjunction, the second one is the denial of one of the conjuncts of that conjunction, and the conclusion is the other conjunct. There are two problems regarding this schema: Firstly, it is not attributed to Stoic logic in any other ancient source. Secondly, as it can be easily checked, it is not logically correct (at least if evaluated following standard propositional logic). In particular, its Latin expression is as follows:

> "*Non et hoc et illud; non hoc; illud igitur*" [i.e., in English, "Not both this and that; not this; therefore that" (translation coming from O'Toole & Jennings 2004, 511)].

As O'Toole and Jennings (2004, 510-512) comment, the usual view on this fact is that Cicero made a great logical mistake. In fact, O'Toole and Jennings (2004, 511)



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remind us that Calvin Normore even proposed that the cause of this clanger was Cicero's insomnia. However, they also describe different alternative interpretations of this fact that enable to think that Cicero did not really make an error and that, on the contrary, he was exposing relevant aspects of Stoic logic. Without adopting necessarily O'Toole and Jennings' (2004) opinion about this case, I will follow this general idea in this paper and assume that Cicero was not wrong. In my view, a very suggestive possibility is to think that Cicero presented a schema that actually was valid in Stoic logic. This possibility is suggestive because, as shown below, it allows us to propose some hypotheses about certain unclear or unknown points in Stoic logic. Of course, a third possibility can be that most of the Stoics did not accept the schema, and that it was the result of Cicero's interpretation (not of his error). Nevertheless, as it can also be checked below, even if this last case is the correct one, it is worth analyzing the schema in order to identify the consequences and the conclusions that can be derived if it is assumed.

To do that, I will resort, only as a methodological tool, to the mental models theory (some recent works supporting this theory can be, for example, Hinterecker, Knauff, & Johnson-Laird 2016; Johnson-Laird 2010, 2012, 2015; Khemlani, Lotstein, Trafton, & Johnson-Laird 2015; Khemlani, Orenes, & Johnson-Laird 2012, 2014; Orenes & Johnson-Laird 2012). This theory is a contemporary theory on human reasoning. Nonetheless, its use in this paper does not mean that I necessarily accept its theses or that I will argue in favor of it. My only intention is to make an analysis of possibilities such as those that are often made by the proponents of the mental models theory in order to discover or identify the exact characteristics of the schema indicated by Cicero, and to check whether or not such characteristics are compatible or consistent with the general frame of Stoic logic. So, as mentioned, only the methodology of analysis of the mental models theory is interesting for the aims of this paper, and not its theses or its view about human inferential activity.

However, it can be opportune to begin with a description of what Cicero's schema is really. For that reason, firstly, I will offer that description. Thus, secondly, I will make an analysis of the possibilities that correspond to the schema similar to those that are usually carried out by the mental models theory for explaining inferences and deductions made by human beings. Because that analysis will reveal certain characteristics of the schema, finally, I will try to check whether or not such characteristics are coherent with the other Stoic schemata and it is possible to state that Stoic logic included one more schema that has so far remained lost.

**Cicero' schema**

The mentioned Cicero's schema seems to be very akin to other five schemata named '*indemonstrables*' (*ἀναπόδεικτοι*) and that are usually attributed to Chrysippus of Soli (e.g., Diogenes Laërtius, *Vitae Philosophorum* 7. 80-81). As it is well known, the formal structure of such schemata is as follows:

-Modus Ponendo Ponens:

p -> q
p
--------
Ergo q

Where '->' stands for conditional relationship.

-Modus Tollendo Tollens:

p -> q
¬q
----------
Ergo ¬p
Where '¬' is denial.

-Modus Tollendo Ponens:

p v q
¬p
--------
Ergo q

Where 'v' denotes disjunction.

-Modus Ponendo Tollens (I):

¬(p · q)
p
----------
Ergo ¬q

Where '·' represents conjunction.

-Modus Ponendo Tollens (II):

p $\underline{v}$ q
p
---------
Ergo ¬q

Where '$\underline{v}$' refers to exclusive disjunction.

As it is also well known, these five schemata are valid in standard propositional calculus and, of course, under Gentzen's (1934, 1935) system. Maybe the only problem can be Modus Ponendo Tollens (II), since in standard propositional calculus disjunctions are inclusive. Nevertheless, this problem is really apparent, as that calculus has a way to express exclusive disjunctions. This way consists of considering p $\underline{v}$ q to be equivalent to (p v q) · ¬(p · q), and, obviously, ¬q can be derived from (p v q) · ¬(p · q) and p in standard logic.

In fact, even another contemporary theory on human reasoning, the mental logic theory (see, e.g., Braine & O'Brien 1998a; O'Brien 2009, 2014; O'Brien & Li 2013; O'Brien & Manfrinati 2010), takes three of them as 'Core Schemata': Modus Ponendo Ponens, Modus Tollendo Ponens, and Modus Ponendo Tollens (I). This

theory claims that human reasoning works in accordance with formal or syntactic rules, but it does not assume all the rules of standard calculus. Following the mental logic theory, people only uses some schemata of that calculus, which are not linked to truth tables. Nonetheless, what is interesting here is that the mental logic theory seems to show us that Chrysippus' *indemonstrables* continue to be relevant at the present time. As said, it clearly accepts only three *indemonstrables*, but it can offer an account of the other two schemata as well. As shown by López-Astorga (2015a: 6-8), it can deal Modus Ponendo Tollens (II) in a similar way as standard propositional calculus, i.e., considering p $\underline{v}$ q to be equivalent to (p v q) · ¬(p · q). Thus, given that the theory assumes Modus Ponendo Tollens (I) and a 'Feeder Schema' to eliminate conjunctions (see Braine and O'Brien 1998b: 80), it enables to derive ¬q from (p v q) · ¬(p · q) and p too. As far as Modus Tollendo Tollens is concerned, apart from the fact that López-Astorga (2015a: 8-12) offers an explanation based on the mental logic theory of the reasons why the Stoics could have assumed it as well, it must be said that the mental logic theory states that is not a basic schema usually applied by people because experimental results reveal that individuals not always consider it. Nonetheless, likewise, the theory says that logically sophisticated individuals can know the needed strategies and have the required skills to use that schema. In particular, the strategy that needs to be considered is Reductio ad Absurdum (see also Braine & O'Brien 1998b: 83).

In any case, our problem is that Cicero claims that Stoic logic also includes another schema. That schema, which has been indicated above, has this formal structure:

¬(p · q)
¬p
---
Ergo q

As said, this schema has two problems: Cicero's *Topica* is the only source in which it is mentioned and it is not valid in standard propositional calculus. In fact, it is not admitted by the mental logic theory either. For these reasons, as also commented, Cicero' schema has been considered as an error made by him.

However, O'Toole and Jennings (2004: 510-512) expose some alternative interpretations of this fact that can lead one to think that Cicero was not actually wrong. One of them is based on the datum that the Stoics understood disjunction, i.e., expressions such as *ἤτοι... ἤ...*, as exclusive (even in Modus Tollendo Ponens). Indeed, as shown by O'Toole and Jennings (2004: 498-450), both ancient sources (e.g., Gellius, *Noctes Atticae* 16.8; Galen, *Institutio Logica* 5.1) and contemporary authors (e.g., Bocheński 1963: 91; Mueller 1978: 16) seem to argue on this point. If this is so, because exclusive interpretation is not the most appropriate

interpretation in ancient Greek and it can be considered to be a technical definition of disjunction in Stoic logic, it can be assumed, without difficulties, that denied conjunctions also had a technical definition in that logic. That definition would state that a denied conjunction is false not only when its two conjuncts are true, but also when they are false. In fact, examples of denied conjunctions whose conjuncts cannot be false at the same time can be easily thought. One of them can be clearly a proposition such as this one:

It is not true that he is alive and he is dead

The reason is evident: If he is not alive, he is dead and, if he is not dead, he is alive.

However, O'Toole and Jennings (2004, 510-512) appear to go beyond these arguments and think about other possibilities too. Thus, it can be said that they seem to state that, if Stoic logic is considered from a formal perspective, it can be proposed that the Stoics adopted the following definitions:

Disjunction = $(p \vee q) \cdot \neg(p \cdot q)$

Denied conjunction = $(p \vee q) \cdot \neg(p \cdot q)$

Therefore, disjunction = denied conjunction [from now on, I will refer to the denied conjunction in which the two conjuncts cannot be false with the expression $\neg(p \underline{:} q)$].

As indicated, the definition of disjunction corresponds to the way in which both standard propositional calculus and the mental logic theory can work with exclusive disjunctions. Thus, what would be different and distinctive of Stoic logic would be its definition of denied conjunction.

Nevertheless, O'Toole and Jennings seem to adopt a different perspective and argue that to consider Stoic logic to be a formal approach is not the most adequate way of understanding this logic. To the contrary, I will continue to analyze in details the possibility that Stoic logic offered a formal framework. What is curious is that, to do that, I need to resort to a semantic approach: the mental models theory.

## The semantic theses of the mental models theory

As said, the use of the mental models theory here is not intended to show that it is a correct theory or that their main theses are right. Undoubtedly, the mental models theory is a powerful theory that, as it can be checked in cognitive science literature, can explain and predict most experimental results in reasoning tasks. Nonetheless, the mental logic theory is also an important theory at present and it is consistent with a lot of experimental results as well. In addition, these theories are not the on-

ly current proposals on human cognition. So, the discussion about how human beings actually reason is fairly broad and complex, and that debate is clearly away from the aims of this paper. For this reason, as also mentioned, I will only resort to the mental models theory as a methodological tool that can help us understand the real characteristics of Cicero's schema and its possible role in Stoic logic.

Having said this, it can be stated that the mental models theory is a semantic approach based on the possibilities, or models, which can be attributed to different propositions or sentences. The theory claims that all models are not similar and that all of them do not have the same status. There are 'mental models' (which are immediately available for individuals) and 'fully explicit models' (which can only be taken into account if individuals make certain effort). But, given that, as indicated, the analysis of possibilities of the mental models theory will be only a methodological instrument here, I will ignore that distinction and focus only on the fully explicit models, since they represent all the possibilities that can be related to a particular proposition.

The mental models theory assigns models to each operator in classical logic. In this way, the fully explicit models of disjunction (i.e., all the possible situations in which disjunction is true) are the following:

| -Inclusive: | | -Exclusive: | |
|---|---|---|---|
| p | q | p | not-q |
| p | not-q | not-p | q |
| not-p | q | | |

As it can be noted, when disjunction is inclusive, the only scenario in which it is false is when both p and q are false (i.e., when both not-p and not-q happen). However, if exclusive, disjunction is also false when both p and q are true (i.e., when both p and q happen).

On the other hand, conjunction has only a fully explicit model:

| | |
|---|---|
| p | q |

That is, a conjunction is only true when both p and q are true (i.e., when both p and q happen).

As far as the conditional is concerned, its fully explicit models are:

| | |
|---|---|
| p | q |
| not-p | q |
| not-p | not-q |

These models remind us the interpretation of the conditional attributed to Philo of Megara (e.g., Sextus Empiricus, *Adversus Mathematicos* 8.245), since, as it can

be noted, the conditional is only false when p is true and q is false (i.e., when p happens and q does not happen). In the case in which p is false and q is true (the second model), the conditional is true.
Finally, the biconditional has these fully explicit models:

p q
not-p not-q

These models reveal us that the biconditional is true when both p and q are true and when both of them are false (i.e., when both p and q happen and when both of them do not happen).

Based on this, the mental models theory explains inferential processes by means of combinations of possibilities. Let us think, for example, about Modus Tollendo Ponens. Its first premise, p v q (or p $\underline{v}$ q), has the models indicated for inclusive (or exclusive) disjunction. Nevertheless, the second premise, ¬p, eliminates the models in which p is true, i.e., the first and the third models (or only the third model if the disjunction is considered to be exclusive). So the result is that, given the premises, only one scenario is possible:

not-p q

Therefore, the conclusion is that, given that not-p is true, the only possibility is that q is true too. In other words, it is not possible a scenario in which not-p is true and q is not.

But two more points about this theory are important for this paper as well. Firstly, the models corresponding to a denied expression are the complement of the models of that expression without denial (see Khemlani et al. 2012, 2014). In this way, if the model of conjunction is the scenario in which both p and q are true, the models of a denied conjunction are:

not-p q
p not-q
not-p not-q

Secondly, although the models sets seem very akin to truth tables of classical logic, they are not truth tables at all. A very important concept of the mental models theory is that of modulation. That concept refers to the fact that some models can be blocked by the action of meaning, context, or pragmatics. Let us consider an inference with these two premises:

If he is a philosopher, then he is not Platonic

He is Platonic

In principle, Modus Tollendo Tollens should lead us to conclude that he is not a philosopher. Nevertheless, most people (at least if they had certain knowledge on ancient philosophy) would not draw that conclusion from the mentioned premises. What would happen would be the following:

The initial fully explicit models of the first premise would be:

| | |
|---|---|
| philosopher | not-Platonic |
| not-philosopher | not-Platonic |
| not-philosopher | Platonic |

Nonetheless, modulation would remove the third model (it is not possible to be Platonic and not to be a philosopher). But, because of the second premise (he is Platonic), the other two models would be also eliminated. Therefore, nothing could be derived from the premises.

However, what is truly interesting about modulation in this paper is that, although the proponents of the mental models theory think that reasoning processes are exclusively semantic and logical forms do not play a role in human inferential activity (as shown, that activity can be explained without resorting to logical forms or syntactic rules), possibilities or models can be linked to truth tables and reveal us underlying logical forms. This idea is explained in papers such as, for example, that of López-Astorga (2014, 2015b) and, based on it, the following argumentation can be proposed:

If the first premise of the previous example is taken into account, it can be said that its only real models are:

| | |
|---|---|
| philosopher | not-Platonic |
| not-philosopher | not-Platonic |

If we consider p to be equivalent to 'philosopher' and q to be equivalent to 'Platonic', it can be stated that what the models show is that the first premise is only true in the cases of $p \cdot \neg q$ and $\neg p \cdot \neg q$. But, as it is well known, in classical logic, there is a truth table that establishes exactly that. That truth table is that of $\neg q$. This means that, based on the analysis of possibilities of the mental models theory, it can be said that the logical form of the premise 'If he is a philosopher, then he is not Platonic' is not p -> ¬q, but simply ¬q (which in turn reveals us that the two premises are incompatible).

The mental models theory would not accept this argument, since, as said, it considers reasoning to be only semantic. Nevertheless, I think that the argument shows a consequence of the theory that is interesting and that can be useful for the goals of this paper. As mentioned, I am not assuming here the theses of the mental models theory. I only want to use its analysis of possibilities in order to

detect what it can show about Cicero's schema. I will do that in the next section and I will mainly base my account on the previous argument, which, as indicated, comes from works such as that of López-Astorga (2014, 2015b).

## The analysis of possibilities and Cicero's schema

In the case of Cicero's schema, it can be said that, given that it is a denied conjunction, its models or possibilities are these ones:

| | |
|---|---|
| not-p | q |
| p | not-q |
| not-p | not-q |

The problem is that the second premise (¬p) only removes the second model and that two scenarios hence are possible: a scenario in which p is false and q is true (first model), and a scenario in which both p and q are false (third model). Because Cicero's conclusion is q, it is clear that he thought that the third model is not possible for denied conclusions. So, it can be argued that, in principle, the analysis of possibilities of the mental models theory reveals that O'Toole and Jennings' (2004) alternative explanations mentioned above are very plausible. Thus, on the one hand, it can be justified to assume that Stoic logic (or Cicero's interpretation of it) understood denied conjunctions in a different way from classical logic and proposed that, when the two conjuncts of a denied conjunction are false, that expression is not true. In this way, taking into account the concept of modulation, it can be thought that Stoics' interpretation of denied conjunctions considers such expressions to be false not only when the two conjuncts are true, but also when both of them are false. This idea can seem odd. Nevertheless, it does not seem so anymore if we pay attention to the fact that the other operators also had a different interpretation from that of standard logic in Stoic logic. As commented, the expressions of the kind *ἤτοι... ἤ...* were always interpreted by the Stoics as exclusive disjunctions and, as it is well known, Chrysippus of Soli is said to reject the classical logic interpretation of the conditional, i.e., the material interpretation, or, if preferred, Philo's interpretation. Indeed, under Chrysippus' interpretation of the conditional, the denial of the consequent (*λῆγον*) has to conflict with the antecedent (*ἡγούμενον*) (see, for example, Diogenes Laërtius, *Vitae Philosophorum* 7.73). Therefore, situations in which the antecedent is false and the consequent is true do not appear to be possible.

Nonetheless, on the other hand, it is also worth considering the possibility raised by O'Toole and Jennings (2004) related to a possible equivalent technical definition for disjunction and denied conjunction. As said, according to this possibility,

$p \underline{v} q = \neg(p \underline{:} q) = (p \vee q) \cdot \neg(p \cdot q)$

These equivalences mean that a possible truth table of denied conjunction in Stoic logic would be equivalent to that of exclusive disjunction. This can be easily noted if the two models that appear to be valid for the premise with a denied conjunction in Cicero's schema are taken into account. Such models are not-p and q, and p and not-q, i.e., the models corresponding to the situations in which, in classical logic, exclusive disjunction is true.

But an interesting point is that $\neg(p \underline{:} q)$ is a denied expression, and that one might suppose that, if the denial is removed, its truth-values must be the opposed values. Thus, if $\neg(p \underline{:} q)$ is true in the cases of $\neg p \cdot q$ and $p \cdot \neg q$, $p \underline{:} q$ should be true in the cases of $p \cdot q$ and $\neg p \cdot \neg q$, which, if it is assumed that '<->' stands for biconditional relationship, in turn means that $p \underline{:} q$ is equivalent to p <-> q (remember that p <-> q is false in the cases of $p \cdot \neg q$ and of $\neg p \cdot q$). And this can lead one to think that what the Stoics (or Cicero's interpretation) proposed was that, as disjunction and the conditional, Stoic conjunction, and not only Stoic denied conjunction, had a different interpretation from that of standard logic. That interpretation was that of the biconditional and assumed that the scenarios in which the two conjuncts are false make conjunction true.

There is no doubt that this interpretation can be consistent even with the Stoics' metaphysical and ethical beliefs. It is absolutely possible that ideas such as that of determinism or that of cosmic order had an influence on their logical theses. In this way, they could think that there are elements in the universe that can only happen together, and the only alternative to that is that neither of them happens. But the problem is that this idea is a simple hypothesis because, as far as I know, no ancient source indicates something similar.

In any case, based on the previous arguments, it can be said that all the previous accounts are clearly linked. If, in a denied conjunction, the two conjuncts cannot be false, it can be stated that $p \underline{v} q = \neg(p \underline{:} q)$. Likewise, given that $p \underline{v} q = \neg(p <-> q)$, it is not hard to understand that conjunction is interpreted as the biconditional either. Therefore, in my view, we have prima facie evidence that allows supposing that, in Stoic logic (or under Cicero's interpretation of that logic), there could be a sixth *indemonstrable*. If this hypothesis is right, Cicero did not make a clanger, Modus Tollendo Ponens could be really Modus Tollendo Ponens (I), and Cicero's schema could be Modus Tollendo Ponens (II).

## Conclusions

What is undoubtedly a mistake is to think that Stoic logic was akin to classical logic (several authors have already held similar ideas, e.g. Bobzien 1996, 134). The Stoics took into account three logical operators: the conditional, disjunction, and

conjunction. It appears to be obvious that the conditional and disjunction were not interpreted as they are in standard logic. Stoic logic rejected Philo's interpretation of the conditional and assumed that of Chrysippus. On the other hand, they understood disjunction as exclusive. Actually, we do not know for sure which their interpretation of conjunction was. However, as shown above, we have evidence that it is very possible that it was interpreted as the biconditional in classical logic. If this is so, Cicero did not make an error. He provided us clues about how the Stoics understood conjunction.

It is also a possibility that Cicero did not describe the real Stoic view of conjunction, but his own interpretation. Nevertheless, the method of analysis of the mental models theory helps us understand that it is plausible that the Stoics thought that, in a denied conjunction, the two conjuncts cannot be false, that both disjunction and denied conjunction mean $(p \vee q) \cdot \neg(p \cdot q)$, that denied conjunction hence is equivalent to exclusive disjunction, and that, therefore, conjunction is equivalent to the biconditional. All this, as said, enables us to think about the possibility of a sixth *indemonstrable*, Modus Tollendo Ponens (II), which would have the formal structure of Cicero's schema.

However, it is necessary to insist on two important points. Firstly, the mental models theory is a contemporary reasoning theory. As said, according to it, only semantics play a role in human reasoning and syntax is not relevant for it. So, the methodological use of the way it analyzes possibilities that I have done in this paper is not accepted by most of its proponents. That use has been proposed in papers such as that of López-Astorga (2014, 2015b), but, as indicated, is not a part of the theory. Secondly, as also mentioned, the idea that the Stoics interpreted conjunction as the biconditional cannot be clearly found in any ancient source. So, it can be thought that this is a simple hypothesis. Nevertheless, it is possible to offer strong arguments in favor of it. One very important point is that several authors and writers have considered some testimonies with regard to other issues given by Cicero to be decisive. For example, a passage in *De Fato* 12-16 has been deemed as a proof that Chrysippus of Soli rejected Philo's interpretation of the conditional and claimed an actual relation between the antecedent and the consequent (as said, the denial of the consequent had to be inconsistent with the antecedent). Some of those authors and writers are, for instance, Gould (1970: 76), Mueller (1978: 20), and O'Toole and Jennings (2004: 479). For this reason, one might question why certain passages authored by Cicero are accepted as credible and others of them are not. A possible answer can be that the inference schema described in *Topica* 13.57 is incorrect and hence reveals that Cicero made a clear mistake. Nevertheless, that schema is only incorrect if it is evaluated from stand-

ard logic. In this way, as explained, there are no evident reasons to doubt the correctness of Cicero's schema.

Thus, if we admit that the schema was an authentic Stoic schema, as argued above, that leads us in turn to accept that, with all probability, the Stoics technically defined conjunction as the biconditional. Furthermore, if we actually have no reasons to think that Cicero was wrong, we have no reasons to think that this 'new' *indemonstrable* can be assigned only to Cicero either. In fact, in view of these arguments, it seems to be absolutely justified to assume that Stoic logic included a sixth *indemonstrable*: Modus Tollendo Ponens (II).

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