Pythagorean Numerology and Diophantus' *Arithmetica* (A Note on Hippolytus' *Elenchos* I 2)

Eugene Afonasin – Novosibirsk State University

Keenness backed by teaching is a swift road to knowledge.
Diophantus, *Arithmetica* I, pref.

1

According to Hippolytus (*Elenchos* I 2, 18),[1] the Pythagoreans "borrowed their number theory and the system of measuring from the Egyptian priests."[2] An outline of Neopythagorean numerology, which follows this statement, contains nothing "Egyptian," of course.[3] But to his otherwise typical summary Hippolytus unexpectedly adds the following extraordinary statement (repeated verbatim in a similar exposition of the Pythagorean doctrine in book IV 51, 8), which, as it turns out, may be of interest to the historians of mathematics:

1 The edition used is that of Miroslav Marcovich (1986). An English translation by J. H. Macmahon in the fifth volume of the *Ante-Nicene Christian Library* (Roberts / Donaldson 1867–1897) and that of F. Legge (1921) are mostly reliable but require corrections in some places. The work can be dated to the beginning of the third century (AD 222–235, according to Marcovich 1986, 17). As regards the character of the author, I prefer to suspend judgment. For details cf. Cerrato 2002 (with my review in *Bryn Mawr Classical Reviews*), Osborne 1987, Mansfeld 1992.

2 Τοὺς δὲ ἀριθμοὺς καὶ τὰ μέτρα παρ' Αἰγυπτίων φασὶ τὸν Πυθαγόραν μαθεῖν· Similar phrases open other summaries of Pythagorean doctrine: in IV 51, 1 Hippolytus simply restates this, while in VI 21, 1–2 he adduces a "testimony" from Plato's *Timaeus* (19e). Clearly the idea is of fundamental importance for him: the Pythagoreans used to lead a solitary life in underground chambers, "being struck by the plausible, fanciful, and not easily revealed wisdom" of the Egyptian priests (*Elenchos* I 2, 16–18). Pythagorean ties with Egypt are a commonplace, although the reason for keeping silence during the period of instruction given by Hippolytus is unusual and perhaps arose as a result of merging two or more separate reports into one succinct testimony (silence = a solitary life).

3 He starts with definitions of the first number (an originating principle, singular, indefinable, incomprehensible, containing in itself all numbers that, according to plurality, can go on ad infinitum) and the primary unit (a principle of numbers, a male entity, a parent of all the rest of the numbers). The dyad (a female even number), the triad (a male uneven number), and the tetrad (a female even number) follow them and produce a decade ("the source of everlasting nature"), etc. This variant of Pythagorean numerology roughly corresponds with the "typical" Pythagorean metaphysics known to have been developed by the members of the Old Academy (Speusippus, Xenocrates, etc.). For detailed analyses of the subject see, for instance, Burkert 1972, 53 ff. and Dillon 2003, 40 ff., 110 ff.

> The four divisions of this decade, the perfect number, are called (καλεῖται) number, unit, *dynamis*, and *cubus* (ἀριθμός, μονάς, δύναμις, κύβος), the conjunctions and minglings of which make for the birth of increase and complete naturally the productive number (τὸν γόνιμον ἀριθμόν). For when the *dynamis* is multiplied by itself, a *dynamodynamis* is the result (ὅταν γὰρ δύναμις αὐτὴ ἐφ› ἑαυτὴν κυβισθῇ, γέγονε δυναμοδύναμις). But when the *dynamis* is multiplied into the *cubus*, the result is a *dynamocubus* (ὅταν δὲ δύναμις ἐπὶ κύβον, γέγονε δυναμόκυβος); and when the *cubus* is multiplied into the *cubus*, the product of two cubes (*cubocubus*) is the result (ὅταν δὲ κύβος ἐπὶ κύβον, γέγονε κυβόκυβος). So that all the numbers from which the production of existing (numbers) arises, are seven – namely, number, unit, *dynamis, cubus, dynamodynamis, dynamocubus, cubocubus* (ἀριθμόν, μονάδα, δύναμιν, κύβον, δυναμοδύναμιν, δυναμόκυβον, κυβόκυβον).

This is an excerpt from Diophantus. In his *Arithmetica* I (Praef., ed. Tannery I, 2, 3 ff.; I. Thomas' translation) this Greek mathematician likewise starts his exposition with definitions of *number* (ἀριθμός) and *unit* (μονάς), "all numbers are made up of some multitude of units" and "their formation has no limit" and proceeds by saying that the square of a determinate number (**τετράγωνον**) is formed when "any number is multiplied by itself; the number itself is called the side of the square (the square root);" cubes are formed when "squares are multiplied by their sides;" square-squares (δυναμοδύναμις) are formed when squares are multiplied by themselves; square-cubes (δυναμόκυβος) are formed when squares are multiplied by the cubes formed from the same side; cubo-cubes (κυβόκυβος) are formed when cubes are multiplied by themselves.[4]

A *Thesaurus Linguae Graecae* (TLG) search helps to identify the place of Hippolytus' source within the Greek literary tradition. Take, for instance, the κυβόκυβος. The TLG gives twelve instances in Diophantus himself (all from the *Arithmetica*), eight in the *Scholia in Diophantum* (all within one passage), one in the *Scholia in librum Iamblichi in Nicomachi arithmeticam introductionem* (with a reference to Diophantus), and, finally, our passage in Hippolytus (three times).

A search for δυναμοδύναμις yields similar results (Diophantus, Hippolytus, *Scholia in Diophantum, Scholia in Iamblichum*) with two interesting exceptions. The term is twice used in the *Metrica* (I 17, lines 17, 25 and 26, ed. H. Schöne) of Heron of Alexandria and once in the Neoplatonic *Anonymous prolegomena to Platonic philosophy* (6, ed. L. G. Westerink), discussed below. So the terminology must have

4 He introduces the following notation: the square of the unknown quantity is called *dynamis* (signified by Δ with index Υ, that is $\Delta^{\Upsilon} = x^2$ in contemporary notation); the cube – *cubus* ($K^{\Upsilon} = x^3$); the square multiplied by itself – *dynamodynamis* ($\Delta^{\Upsilon}\Delta = x^4$); the square multiplied by the cube formed from the same root – *dynamocubus* ($\Delta K^{\Upsilon} = x^5$); the cube multiplied by itself – *cubocubus* ($K^{\Upsilon}K = x^6$). Fractions are defined similarly: *arithmoston* – 1/x; *dynamoston* – $1/x^2$; *cuboston* – $1/x^3$, etc. He also introduces special marks for number, unit, and unknown value.

originated in the works of such mathematicians as Heron and Diophantus and, for some reason, aroused interest only in Platonic and Pythagorean circles. Indeed, if we were to look for an example of an "ideal" Pythagorean, Diophantus would certainly qualify: he wrote a book on such a popular Pythagorean subject as polygonal numbers, transformed traditional arithmetic, and created a new theory of number; in a word, he did things a Pythagorean is supposed to do. It is no secret that Pythagorean numerology is not very useful from a mathematical point of view. What if an unknown Pythagorean (Hippolytus' source) decided to translate the standard Pythagorean theory into the language of the higher mathematics of his times? It is as if asked to define number I were to indulge in axiomatic set theory.

The anonymous author of the *Prolegomena to Platonic philosophy* comments on the fact that Plato died at the age of 81: this number is associated with Apollo and Muses, he says, and bi-quadratic (δυναμοδύναμις), because the number three[5] multiplied by itself gives nine, while nine multiplied by itself gives our 81, i. e. $3^2 \cdot 3^2 = 3^4 = 81$. We know nothing about Diophantus' life, but he may not have been a complete stranger to Pythagorean circles, as a mathematical riddle, allegedly an inscription on his tomb, which tells scientifically the span of his life, seems to attest (*Palatine Anthology* XIV 126).[6]

It is intriguing that Hippolytus is not alone in ascribing this sort of mathematics to the Egyptians. A Byzantine polymath Michael Psellus (Thomas II.514–515) also says that Diophantus "more accurately" developed a certain "Egyptian method." Compare a *Scholium to Plato's Charmides 165e* (Thomas I.17) where logistic is defined as "a science that treats of numbered objects, not of numbers" (1 as a unit, 3 as a triad, 10 as a dyad, etc.). It deals, according to the scholiast, with such problems as Archimedes' *Cattle Problem* and

> its branches include the so-called Greek and Egyptian methods of multiplication and division, as well as the addition and splitting up of fractions,

5 "The first number which contains in itself the beginning, the end, and the middle;" compare with Anatolius *ap.* the *Theologoumena arithmeticae* 17.

6 "Here lies Diophantus. The wonder behold – Through art algebraic, the stone tells how old: 'God gave him his boyhood one-sixth of his life, / One-twelfth more as youth while whiskers grew rife; / And then yet one-seventh ere marriage begun; / In five years there came a bouncing new son. / Alas, the dear child of master and sage / After attaining half the measure of his father's life / Chill fate took him. After consoling his fate by the science of numbers / For four years, he ended his life'" (trans. W. R. Paton). Let x be the numbers of years Diophantus lived, then, given that the number of years his son lived is $x / 2$, the problem is solved thus: $x = x / 6 + x / 12 + x / 7 + 5 + x / 2 + 4$ (quoted according to Eric Weisstein, "Diophantus' Riddle:" mathworld.wolfram.com / DiophantussRiddle.html). The solution is 84 years, admittedly, not as perfect as in the case of Plato, but still interesting: consider, for instance: $3 + 81 = 3 + 3^4 = 3 \cdot (1+3^3)$. Cf. "Pythagorean" methods of finding "side- and diameter-numbers" (Thomas I.90–95, 132–139, examples from Aristotle, Lucian, Nicomachus, Theon of Smyrna, and Proclus), Archimedes' famous *Cattle Problem* (Thomas II.202–205), Heron's problems (II.504–509), etc. Compare also the *Theologoumena arithmeticae* 52–53, where Anatolius applies 6^3 as part of the calculation of Pythagoras' lifespan: the answer is 82. We will return to this later.

> whereby it explores the secrets lurking in the subject-matter of the problems by means of the theory of triangular and polygonal numbers. Its aim is to provide a common ground in the relations of life and to be useful in making contracts, but it appears to regard sensible objects as though they were absolute. (trans. Thomas).

Speaking about the Egyptian influence, our sources do not allude to anything especially mystical, nor ascribe to Pythagoras any sort of secret Egyptian lore: the ancient testimony is in agreement with the historical truth, as the Egyptian papyri amply testify (cf. Fowler / Turner 1983, Knorr 1982, 140 ff., Chrisomalis 2003).

2

History was not kind to Diophantus: valued nowadays as one of the greatest mathematicians of all times, he was almost completely forgotten by his contemporaries.[7] We know nothing about his life, and his origin is also uncertain. The fundamental *Arithmetica* (now supplemented by the Arabic books, edited in 1982–1984) definitely belong to Diophantus; a partially preserved treatise *On Polygonal Numbers* is traditionally ascribed to him. A collection of propositions in the theory of numbers, called the *Porisms*, is several times mentioned by the author in Book III of his *Arithmetica*. Jean Christianidis (1980) suggested that certain (lost to us) "elements of arithmetic" (Ἀριθμητικὴ στοίχειωσις), ascribed to Diophantus by an anonymous Byzantine commentator to Iamblichus' *Introduction to Nicomachus' Arithmetic* (132, 10–13 Pistelli),[8] could have served as an elementary introduction (similar or identical with this *Porisms*). This is interesting because, transmitted independently, this introductory work would be able live its own life and, unlike the exceptionally difficult major treatise of Diophantus, could have influenced popular arithmetic; such popular arithmetic took forms such as elementary textbooks (Robbins 1929; Fowler 1983), mathematical epigrams, like those collected

7 The *status questionis* is well summarized by N. Schappacher (2005, 9 ff.): "The *Arithmetica* are almost as elusive as their author. The way in which mathematicians through the centuries have read and used the *Arithmetica* is always a reliable expression of their proper ideas; but never can we be sure of what these readings tell us about the text itself. The *Arithmetica* are probably the most striking example of a mathematical text which, on the one hand, has inspired, and continues to inspire, generations of mathematicians at various different moments of the history of algebra and number theory; but which, on the other hand, has never, for all that we know, been developed further as such." He then describes "four major renaissances of Diophantus, or, more precisely, two times two" which occurred between the ninth and thirteenth centuries in the world of Islam and Byzantium and between the sixteen and seventeenth centuries in Europe and the Western world. The four Arabic books of Diophantus were discovered in the 1970s and the Greek mathematician still inspires contemporary mathematical thought (28 ff.).

8 The text reads: "We shall learn the properties of the harmonic mean more fully in the final theorem of the first book of Diophantus' presentation of the elements of arithmetic, and the diligent should read them there" (132 Pistelli, Waterhouse's translation).

by Metrodorus (who, by the way, calls Diophantus' work στοιχεῖα[9]), and the Pythagorean numerological works similar to this one excerpted by Hippolytus. This attribution was somewhat undermined by W. Waterhouse (1993), who, following Heath, argued that the *Elements of Arithmetic* is just another title for the *Arithmetica,* having demonstrated that a "diligent reader" could indeed learn about the geometric mean from the last problem of the first book of the *Arithmetica* (I 39). Despite this, in the same year Wilbur Knorr (1993) not only found additional indications in favour of this attribution,[10] but also argued that another introductory work, the *Definitions*, commonly attributed to Heron of Alexandria, could in fact belong to Diophantus.[11]

Hippolytus' text can clearly serve as a good and virtually neglected *terminus ante quem* for the dating of Diophantus. Largely forgotten in antiquity, Diophantus is for the first time mentioned by name in Theon of Alexandria's *Commentary on Ptolemy's Syntaxis* I, 10 (Thomas II.514).[12] On the other hand, in the work *On*

9 Tannery 1893–1895, II, 62–63, 69. These epigrams correspond to the second problem of *Arithmetica* I.

10 Most importantly, he noticed that ἐν τοῖς πρὸ τῆς ἀριθμητικῆς στοιχειώσεως in the Heronian *Definitions* (128; Heiberg 84.17–19) may actually refer to some *Preliminaries* to the *Elements of Arithmetic*, and not to the work as such. This proves that the *Elements of Arithmetic* could indeed be an alternative title for the *Arithmetica* and, at the same time, leaves open the question of a hypothetical introductory work. His (and Tannery's) guess that, "the Iamblichus scholiast appears to have conflated" two works, "*Preliminaries* being a commentary (presumably in the margins of Diophantus' *Arithmetica*)" (Knorr 1993, 182) is hopelessly conjectural, however.

11 The crucial passage is: Καὶ τὰ μὲν πρὸτῆς γεωμετρικῆς στοιχειώσεως τεχνολογούμεν αὑπογράφων σοι καὶ ὑποτυπούμενος, ὡς ἔχει μάλιστα συντόμως, Διονύσιε λαμπρότατε, τήν τε ἀρχὴν καὶ τὴν ὅλην σύνταξιν ποιήσομαι κατὰ τὴν τοῦ Εὐκλείδου τοῦ στοιχειωτοῦ τῆς ἐν γεωμετρίᾳ θεωρίας διδασκαλίαν – "Also the systematized *Preliminaries* of the *Elements of Geometry*, by writing them below for you and sketching them out, in the most succinct manner, most illustrious Dionysius, I shall make both the foundation and the whole arrangement in accordance with the teaching in the theory of geometry of Euclid, the Elementator" (the opening phrase of the *Definitions*; Knorr's translation, slightly adapted). Knorr observes that this preface and the preface to the *Arithmetica* are addressed to individuals with the same name, Dionysius, and that, in general, they are very similar in their tone, syntax, and style. On the other hand, the *Definitions* are isolated in the Heronian corpus, their attribution to Heron has long been questioned, and, most importantly, the prefaces to other works of Heron are conspicuously different in style and form. This scenario entails, according to Knorr (1993, 186–187), that the *Preliminaries*, called to provide assistance for relative beginners in mathematics, were published by Diophantus after the *Elements of Arithmetic* (= our *Arithmetica*) and "the conspicuous parallelism of the titles – the *Elements of Arithmetic* and the *Elements of Geometry*, by which he refers, respectively, to his own *Arithmetica* and the *Elements* of Euclid, indicates his arrangement of the two great treatises as counterparts within the mathematical curriculum" (Knorr 1993, 188). I find this scenario plausible, with the reservation that the *Preliminaries* could actually have been prepared and published any time before, after, or during the composition of the major treatise.

12 In his fairly elementary work Theon quotes the following statement of Diophantus: "The unit (*monas*) being multiplied without dimensions and everywhere the same, a term (*eidos*) that is multiplied by it will remain the same term." Theon saw the eclipse of AD 364. His daughter Hypatia, also a mathematician, was murdered by the Christians in 415.

Polygonal Numbers, Diophantus quotes Hypsicles (ed. Tannery I 470.27; Thomas II.514), a mathematician and astronomer dated to the mid-second century BC and famous for his division of the circle of the zodiac into 360 parts.[13] Finally, in the *Letter* mentioned above Psellus states that, "the most learned Anatolius collected the most essential parts of the theory as stated by him [Diophantus] in a different way and in the most concise form, and dedicated his work to Diophantus" (Tannery II.38.22–39.1; Thomas II.514).[14] On the basis of these data Tannery concluded that Diophantus must have been a contemporary of Anatolius, whom he identified with a professor of Aristotelian philosophy in Alexandria who was made bishop of Laodicea around AD 270–280.[15] Scholars have doubted this inference and have felt themselves "at liberty to place Diophantus wherever he best fits their theories of historical development" (Swift 1956, 163): some tend to date him closer to the time of Theon of Alexandria, while others relate him to Heron of Alexandria.[16]

13 Hypsicles, *On risings*; Thomas II.394–5. The division of the ecliptic into 360 degrees was, probably, developed by the Egyptians on the basis of the Mesopotamian methods already in the sixth–fifth centuries BC, when Egypt was under the administration of the Persians (Knorr 1982, 157, with a reference to Parker 1972). Hypsicles, however, applied the division to any circle in general. He made a general definition of polygonal numbers, which is quoted by Diophantus. He is also the author of the so-called Book XIV of Euclid's *Elements*, where he refers to Apollonius of Perga as a contemporary.

14 Knorr's (1993, 184–185) suggestion to read *heteroi* instead of *heteros* as in Tannery (which would imply Psellus' reference to "a different Diophantus") further complicates the situation while offering no useful solution. Admittedly, dynasties of intellectuals were common in late antiquity. Take, for instance, Theon and Hypatia (above), or Iamblichus the Younger (the second part of the fourth century), a relative of Sopater, who was a student of the Neoplatonic philosopher Iamblichus (Cameron 1967). Diophantus is not a very common name, but nor is it unique. For instance, a series of superb inscriptions from Lydae (in Lycia, on the southern coast of Turkey) honour a family using this name, founded by a certain Caius Julius Heliodorus, probably a freedman of the Dictator Caesar, which included numerous local officials named Diophantus, and a senator Diophantus, a *rhetor* Heliodorus, etc. An inscription found in the same group honours a physician of Lydae, Ameinias Aristobulus, a man of learning and of distinguished skills, who may belong to the same family (Hicks 1889, 58–75); he is not recorded elsewhere in the literary sources, but Galen mentions a certain Diophantus, also a physician from Lycia (in *De compositione medicamentorum secundum locos* XII 845.8 Kühn). For more details see *Tituli Asiae Minoris* vol. 2, fasc. 1 (1920), nos. 129–157.

15 This Anatolius, mentioned by Eusebius (*Hist. Eccles.* II 726, 6–9), composed ten books of *Arithmetical Introductions*. On the other hand, Eunapius (*Vita Soph.* 363) informs us that a certain Anatolius, a contemporary of Plotinus and Porphyry, was a teacher of Iamblichus. If we, with D. O'Meara (1990, 23), do not wish to multiply Anatolii beyond necessity, and accept an earlier date for Iamblichus' birth (*before* AD 245, according to J. Dillon's suggestion), we can entertain the possibility that Iamblichus studied under Anatolius before the latter became bishop. On the same principle, we may presume that the fragments of a numerological treatise found, along with the excerpts from Nicomachus, in the anonymous *Theologoumena arithmeticae* also belong to him.

16 Knorr (1993), discussed above. Thanks to O. Neugebauer we know that the eclipse of the moon described by Heron (*Dioptra* 35) occurred in AD 62.

I assume that this is the ground for the new dating of Diophantus somewhat earlier than it was previously thought. How much earlier? Our passage contains no indications. But let us look attentively at what follows it:[17]

> [Pythagoras] likewise said that the soul is immortal, and that it subsists in successive bodies (μετενσωμάτωσιν).[18] Wherefore he asserted that before the Trojan era he was Aethalides, and during the Trojan epoch Euphorbus, and subsequent to this Hermotimus of Samos, and after him Pyrrhus of Delos; fifth, Pythagoras.

This was probably invented by a student of Plato and Aristotle, namely Heraclides of Pontus (ca. 380–310 BC).[19] In a passage from Anatolius in the *Theologoumena arithmeticae* 52–53 another student of Aristotle, Aristoxenus (ca. 370–300 BC), and the whole group of Hellenistic historians are credited with a story about Pythagoras' journey from Egypt to Persia as a prisoner of Cambyses (II). The chronology is then used to count the years of Pythagoras' life: the numerologist says that Pythagoras' reincarnations occurred every 6^3 = 216 years; 514 years had passed from the Trojan epoch to the times of Polycrates of Samos; Cambyses was a contemporary of Polycrates; therefore 514-216-216 = 82. Another numerological riddle, and the cubes again!

> But Diodorus of Eretria and Aristoxenus the musician, – continues Hippolytus – assert that Pythagoras went to Zaratas the Chaldaean (Ζαράταν τὸν Χαλδαῖον),[20] and that he explained to him that there are two original causes of things, father and mother, and that father is light, but mother darkness…

Zaratas is then credited not only with distinctly Zoroastrian ideas, but also with the concept of the cosmos as a musical attunement (μουσικὴν ἁρμονίαν), and even with the Pythagorean ban on eating beans, subsequently related to the idea of reincarnation.[21]

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17 *Elenchos* I 2, 12 f. Translations in Kingsley (1990) and Osborne (1987) have been consulted.

18 Cf. VI 26. *Metensomatosis* is a rare word. It occurs once in Clement of Alexandria (τὸ περὶ τὴν μετενσωμάτωσιν τῆς ψυχῆς δόγμα; *Stromateis* 6.35.1.4, where the Indian philosophers are accused of borrowing their doctrines from the Egyptians), once in the Platonic Celsus (*ap.* Origenes, *Contra Celsum* 7.32.12), six times in Hippolytus and from time to time in later literature, most notably, in Origenes (some 20 times), Theodoretus (6 times), Epiphanius (8 times) and other Christian heresiologists and, quite independently of them in Plotinus (twice), Proclus (once), Hermias (once) and Olympiodorus (3 times). One may also note Nemesius of Emesa (5 times). A more standard term would be *metempsychosis*, known at least from the first century BC (Diodorus Siculus 10.5.2.8, a pseudo-Pythagorean source, the so-called *Anonymous Diodori*); also note a pseudo-Pythagorean *Anonymous Photii* and *Theologoumena Arithm.* 52.10 (Aristoxenus, fr. 12.3 Wehrli).

19 Pythagoras had allegedly recognized the shield of Euphorbus, a Trojan hero, killed by Menelaus (cf. Ovid, *Metamorphoses* 15, 160–164, D. L. 8.4 = fr. 89 Wehrli).

20 Cf. VI 23, 2: Ζαράτας ὁ Πυθαγόρου διδάσκαλος.

21 This is truly exceptional. A later tradition about Pythagoras adds to the list of countries he visited the land of the Brahmans, but no one in antiquity ever connected the Pythagorean

This passage, maddening as it stands, has been variously interpreted by scholars. Some, following Eduard Zeller, emphatically denied Aristoxenus' authorship,[22] others, and on good grounds, took a more balanced position.[23] It is true that Aristoxenus is known to have denied that Pythagoras opposed the eating of beans.[24] But this is not decisive in this context. Given that Aristoxenus wrote a great number of works, including other treatises on Pythagoreanism, of which we possess only a handful of fragments and secondary reports, we cannot rule out that he related this somewhere else. Besides, Aristoxenus is famous for his stories about personal contacts between philosophers.[25]

Another solution suggests itself: the list of previous lives of Pythagoras (as seen above) and the peculiar experiment with beans in Hippolytus could be traced back to Heraclides of Pontus.[26] Further, it is known that Heraclides published a (lost) dialogue, entitled "Zoroaster" (Plutarch, *Adv. Col.* 1114F–115A). Could he also have fathered other parts of this report, or, at least have contributed to its development?

We further note that Hippolytus' strange idea that the Pythagoreans led a solitary life in underground chambers, "being struck by the plausible, fanciful, and not easily revealed wisdom" of the Egyptians (*Elenchos* I 2, 16–18), as mentioned

psychology with India, although at least from the Hellenistic period Greek and Latin authors possessed reasonably reliable information about this country. If we are not surprised to find Greek golden pendants in a Hunnic tumulus in Mongolia (Polos'mak *et al.* 2011, 111 ff.), why we are so reluctant to accept the possibility of intellectual contacts between the Greeks and other nations? The Greeks and Indians could have interacted in Persia, just as the Persians did with the Greeks in Egypt. See also Bernabé / Mendoza (2013, 48–49).

22 Most recently: Zhmud 2012, 83 ff.

23 See Kingsley 1990, 246 ff.

24 Aulus Gellius, *Noctes Atticae* 1–4.11 = 25 Wehrli; D. L. 8.20 = fr. 29a Wehrli. And, in general, Aristoxenus differs a great deal from other authors known to have written on Pythagoras. For instance, he is recorded to have established an alternative tradition about Pythagoras' birthplace (Clement, *Stromateis* I 62, 2–3; D. L. 8.1 = fr. 11a Wehrli: Pythagoras was "a Tyrrhenian from one of the islands which the Athenians held after expelling the Tyrrhenians"). It worth noticing that Hippolytus knows about this alternative tradition, as the opening sentence of his report clearly indicates: "*Some* say that Pythagoras was a native of Samos" (Hippolytus, *Elenchos* I 2, 1). Apparently Aristoxenus' opinion was also recorded immediately before this phrase in his source.

25 Kingsley 1990, 252 f. In a recent article Lacrosse (2007) notes that the fictitious discussion between Socrates and the Indian (Eusebius, *Praep. Evang.* I 1, 3 = fr. 53 Wehrli) "echoes the genuine and typically Indian axiom that knowledge of the human self is knowledge of God and vice versa, which is one of the major commonplaces in traditional Brahmanic thought" and "Aristoxenus' fragment is one of the first and only texts, historically, in which a typical Greek philosophical argument is challenged by an authentic Indian proposition translated into an argument based on Greek conceptual categories." Compare the story about Aristotle and the Jewish Sage told by Clearchus of Soli (Lewy 1938), and a passage in the *Theologoumena arithmeticae* 52–53 (discussed above). Even if numerology in the latter is due to Anatolius, the historical details could go back to Aristoxenus.

26 Diogenes Laertius 8.4–5 and Lydus, *De mens.* 99.17, respectively; fr. 41 Wehrli; Marcovich 1968, 32.

above, may be inspired by still another student of Aristotle, Dicaearchus (Porphyry, *VP* 18 = fr. 33 Wehrli).

Finally, a story about the ways new members were accepted into the Pythagorean community, recorded almost immediately after our fragment (*Elenchos* I 2, 16–18), is clearly based on a report found for the first time in a Hellenistic historian Timaeus of Tauromenium (c. 350–260 BC). Pythagoras, allegedly, asked prospective disciples to sell their property and deposit the money with him for the period of instruction. In the event of success, the accepted candidates would hold their property in common,[27] while those rejected would receive their money back. This tradition is relatively early and is usually considered more-or-less credible, as is Hippolytus' repeated statement that the Pythagorean school[28] consisted of two groups of disciples: the insiders ("Esoteric Pythagoreans"), and the outsiders (the Exoterics, also called Pythagoristae).[29] This is the sort of statement one can readily believe, unlike the later tradition about the *mathematikoi* (philosophers and scientists) and *akousmatikoi* (those who receive ethical maxims in a "symbolic" manner), found for the first time in Clement of Alexandria (*Strom*. V 59, 1) and fully developed by Iamblichus.[30] It appears therefore that all the information given by Hippolytus is in its substance traceable back to the Lyceum.

The earliest author known to use this material is Alexander Polyhistor (early first century BC).[31] The source excerpted is also similar to the one appropriated

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27 "What belongs to friends is common property," κοινὰ τὰ τῶν φίλων; Timaeus, fr. 13a Jacoby; Schol. in Plato's *Phaedrus* 279c.

28 Called αἵρεσις (l.4). Compare *Ref.* I 22, 23 and 24.1 where the same term characterizes the Epicureans, the Academics and even the Brahmans. It is safe, therefore, to assume with Mansfeld (1992, 11) that Hippolytus mechanically copied it from his source, rather than introduced it himself. Clement also thought it was typical of any school: the Academics, the Epicureans, the Stoics, and even "the followers of Aristotle say that some of the works of their teacher are esoteric, while the rest is popular and exoteric" (*Strom*. V 58, 1–2; cf. 59, 2).

29 Τοὺς μὲν ἐσωτερικούς, τοὺς δὲ ἐξωτερικοὺς (*Ref.* I 2, 4); οἱ μὲν οὖν ἐσωτερικοὶ ἐκαλοῦντο Πυθαγόρειοι, οἱ δὲ ἕτεροι Πυθαγορισταί (*Ref.* I 2, 17).

30 This well-known subject cannot be treated here. Lengthy discussions are found in Burkert 1972, 192–208 and, recently, Zhmud 2012, 169–206; specifically for Clement's reinterpretation of the concept as a good example of a profound change of attitude to Pythagoras and his school, which took place in the process of transition from the Late Hellenistic to Early Roman period, cf. Afonasin 2012, 27–32.

31 "Pythagoras was enthusiastic about Zoroaster, the Persian Magus, and the followers of Prodicus' heretical sect claim to have obtained secret books of this writer. Alexander, in his work *On Pythagorean Symbols*, records that Pythagoras was a pupil of the Assyrian Zaratas (whom some identify with Ezekiel, wrongly, as I shall show presently), and claims in addition that Pythagoras learned from Gauls and Brahmans (Clement, *Strom*. I 69, 6–70, 1; J. Ferguson's translation)." This, by the way, indicates that the ancients typically distinguished this Zaratas from the prophet Zoroaster, placed in time immemorial (cf. Aristotle, fr. 6 and 34), and routinely cited in this capacity, cf. Clement, *Strom*. I 133, 1 (Zoroaster in a list of real and legendary persons), III 48, 3 (on the Magi in general), V 103, 2 (where Er from Plato's *Republic* is identified with Zoroaster); Hippolytus, *Ref.* V 14, 8 (quoting from a phantasmagoric Gnostic book). Plutarch, *On the Generation of the soul in the Timaeus* first mentions our Zaratas as the teacher of Pythagoras (1012e) and then (1026b) refers to Zoroaster, the author of a

by Antonius Diogenes (ca. AD 100–130).[32] In his *On the Generation of the soul in the Timaeus* (1012e) Plutarch openly admits that he uses an indirect source and then says that Xenocrates (fr. 68 Heinze)

> insert[ed] a limit in infinitude, which they call indefinite dyad (this Zaratas, too, the teacher of Pythagoras, called mother of number; and the one he called father, which is also why he held those numbers to be better that resemble the monad)… (trans. H. Cherniss).

Having combined this testimony with Hippolytus, Harold Cherniss (1976, 165 note C, with reference to Roeper, 1852, 532–535) concludes that behind an otherwise unknown Diodorus (of Eretria) may "lurk" the name of the Neopythagorean philosopher Eudorus (late first century BC), cited several times and frequently used by Plutarch. This well may be the case and Eudorus could indeed have transmitted this information to later writers.

It is safe to assume therefore that our story about Zaratas (if not the entire report) had already been a part of Hellenistic doxographic tradition, reflecting a general tendency to find suitable foreign teachers for all Greek authorities. In the same vein Clement informs us that the teacher of Pythagoras was a certain Sonchis, the highest prophet of the Egyptians, while Plato was associated with a certain Sechnuphis of Heliopolis, Eudoxus the Cnidian studied under Chonuphis, and Democritus spent eight years with certain "Arpedonaptae" (land-surveyors) (*Strom*. I 69, 1 f.). The source of this cento in Clement is unknown, but can probably also be traced to Hellenistic doxography.[33] And, in general, the material analysed seems to indicate that Hippolytus utilized sources that can be dated to a relatively early period. He gives a list of Pythagorean symbols elsewhere (*Elenchos* VI 51, 27, etc), but, as we have seen, knows nothing about the akousmatics and mathematics. This may indirectly indicate that his source(s) were not influ-

mythos about Oromasdes and Areimanius, who (according to his *On Isis and Osiris* 369d–e) lived 5000 years before the Trojan War. Cf. also Diogenes Laertius I 2 (from Hermodorus), Pliny, *Nat. Hist.* XXX 4 (from Hermippus), and, finally, Pletho in Anastos (1948, 280 f.). Be he Zaratas, Zarathustra, or Zoroaster, our authors clearly distinguish between the ancient prophet and the alleged teacher of the historical Pythagoras: a Persian follower of Zoroastrianism could easily have been named after the ancient prophet. It is true, however, that the later Persian and Arabic authors used Aristoxenus' dating of Zoroaster to the sixth century, as Kingsley (1990, 260) has perfectly demonstrated. Pythagoras, according to the Apollodorian system, reached his acme at the time of Polycrates' tyranny in Samos (which means that he was forty ca. 532–529 BC). He was born in 570 therefore, and this date was assumed by the medieval Arabic authors when they calculated that Zoroaster "had appeared" 258 years before the Seleucid era. Cf. a similar numerological exercise above (Anatolius *ap.* the *Theologoumena arithmeticae* 52–53).

32 For a comparative study of the parallel versions of this report in Porphyry, *VP* 44, Lydus, *De mens.* IV 42, and Hippolytus, *Ref.* I 2, 14–15, cf. Marcovich 1964, 29–36. Antonius Diogenes authored a novel, entitled the *Wonders beyond Thule,* now available only as a summary in Photius, in which he claims that he has ancient sources for most of his material, but admits that the work as such is his own literary creation (Morgan 1985, 482). Cf. also Fauth 1978.

33 Diogenes Laertius (8.90) also calls Chonuphis the teacher of Eudoxus.

enced by Neopythagorean biography, which is clearly reflected in such authors as Clement, Porphyry, or Iamblichus. And all the texts exude references to Egypt: Pythagoras studied in Egypt, he borrowed his mathematics and number theory from the Egyptians, and the archetype for the organization of his school is provided by the Egyptian temples.

3

This has clear implications for the dating of Diophantus. If Anatolius, referred to by Psellus, is indeed the future bishop of Laodicea (after AD 270) and Dionysius to be identified with the leader of the Alexandrian 'Catechetical School' (ca. AD 240), then, given that the *Elenchos* was composed before 235, Diophantus must, at least, be an older contemporary of Dionysius, which still places him a generation or two earlier than is traditionally supposed. But, given the nature of Hippolytus' work, it is hard to believe that the idea to interpolate an otherwise traditional Pythagorean text with this piece of "advanced mathematics" occurred to our doxographer. The whole text, as we have seen, unmistakably belongs to early Neopythagorean tradition, which allows us to entertain the idea that Diophantus the *philosophus Pythagoricus*, was known and used in the Neopythagorean and Platonic sources from the second, and, possibly, the first century AD.[34] Moreover, if he, as it seems, authored an introductory work on arithmetic, he may be set in a series with such persons as Eudorus, Cleomedes, Moderatus, Nicomachus, and Theon of Smyrna as a fully fledged contributor to the development of the Neopythagorean movement, perhaps to be placed somewhere between Eudorus and Nicomachus.

An interesting supplementary testimony, which seems to confirm this hypothesis, is furnished by Papyrus Michigan 620, dated to the third century AD. The papyrus contains a series of arithmetical problems "written," according to F. Robbins (1929), "*prior to the time of Diophantus*, in which a quasi-algebraic method of solution is employed and in which appear some of the symbols used in the manuscripts of Diophantus, notably the sign S, which denotes the unknown term." Robbins concludes that, "neither definitely utilitarian, nor so scientifically generalized as the *Arithmetica* of Diophantus… it is most probably a schoolbook of some sort, and perhaps from it or others like it Diophantus may have derived ideas which served as a basis for his mathematical methods." If we now reverse the perspective, could we speculate that this and similar textbooks were in fact influenced by earlier arithmetical works, of which Diophantus' *Arithmetica* was the most advanced example?

34 Knorr also speculates that Diophantus "lived earlier than the third century, possibly even earlier than Hero in the first century" (1993, 187). I was about to complete this essay when I came across the article by Knorr. Is it not significant that two scholars came to a similar conclusion when approaching the problem from completely different perspectives?

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