

WHY PROTAGORAS GETS PAID ANYWAY: A PRACTICAL SOLUTION OF THE PARADOX OF COURT

ELENA LISANYUK

St. Petersburg State University (Russia)

e.lisanuk@spbu.ru

ABSTRACT. The famous dispute between Protagoras and Euathlus concerning Protagoras's tuition fee reportedly owed to him by Euathlus is solved on the basis of practical argumentation concerning actions. The dispute is widely viewed as a kind of a logical paradox, and I show that such treating arises due to the double confusion in the dispute narrative. The linguistic expressions used to refer to Protagoras's, Euathlus's and the jurors' actions are confused with these actions themselves. The other confusion is the collision between the pairs of incompatible actions ambiguously expressed by two different pairs of sentences, one of which is a propositionally consistent pair whereas the other is an inconsistent one. The actional (practical) paradox solution aims to clear up these confusions by means of two core borderlines, propositional and expressive, drawn between the actions and the propositions. The propositional distinction says that actions are empirical facts and they lack truth values unlike propositions, which are mental entities and are often employed for referring to the actions. This distinction helps to avoid the confusion between the empirical incompatibility of actions and the truth-functional inconsistency of propositions. The expressive distinction claims that although the same linguistic sentences can be used to refer both to actions and propositions, two empirically incompatible actions can be expressed both by a pair of inconsistent propositions as well as by a pair of consistent ones. Therefore, the action of Protagoras's being paid may be linguistically symbolized in four different ways: *Protagoras gets paid due to the verdict*, *Protagoras gets paid due to the contract* that amount to *Protagoras does not get paid by the contract* and *Protagoras does not get paid by the verdict* respectively, and likewise for Euathlus's actions. The two distinctions are used for classifying the two groups of paradox solutions, legal and logical, proposed so far depending on which of the two confusions they purport to escape from. The actional reconstruction of the paradox suggests that there is only one single agent in the dispute, Protagoras, while the other named Euathlus is a 'phantom,' which most probably was invented by Protagoras himself for the sake of creating this challenging sophism.

KEYWORDS: Protagoras, logical paradox, argumentation, agency, action, sophism, consistency.

* The support from the Russian foundation for Humanities (project # 15-23-01002) is kindly recognized.

1. Introduction: the story and the paradox

Several ancient historical sources tell a story which is now known as Paradox of court, or Euathlus paradox.¹ The story is as follows:

The famous sophist Protagoras took on a pupil, Euathlus, on the understanding that the student will pay Protagoras for his instruction after he wins his first court case. After instruction, Euathlus decided to not enter the profession of law, and Protagoras decided to sue Euathlus for the amount owed.

Protagoras argued that if he won the case he would be paid his money. If Euathlus won the case, Protagoras would still be paid according to the original contract, because Euathlus would have won his first case.

Euathlus, however, claimed that if he won, then by the court's decision he would not have to pay Protagoras. If, on the other hand, Protagoras won, then Euathlus would still not have won a case and would therefore not be obliged to pay.

The case has embarrassed the jurors to such a degree that they refrained from taking a decision, as Aulus Gellius reports in *Attic Nights*:

Then the jurors, thinking that the plea on both sides was uncertain and insoluble, for fear that their decision, for whichever side it was rendered, might annul itself, left the matter undecided and postponed the case to a distant day.

We will examine how the jurors' escaping from the explicit support of either of the conflict parties' claim in their 'no verdict' for the case is connected to what constitutes the paradox.

Let us take a closer look at where the paradox is stemming from. There are two challenges that constitute the core subject matter of the dispute between Protagoras and Euathlus:

- (i) what the jurors ought to decide?
- (ii) how Protagoras can get paid for his instruction of Euathlus?

We regard (i) and (ii) as challenges and not just as dispute or paradox issues because satisfying them requires both providing a certain reply and a warranting justification for it. It is easy to see that in the paradox story the replies to (ii) stipulate the reasons for the replies to (i). In what follows we will show that the paradox arises in consequence of two confusions: the minor confusion of the roles (i)

¹ The story survives in the three sources: Aulus Gellius's *Attic Nights* (Rolfe 1927, 5.10); Quintilian's *Institutes of the Orator* (Butler 1921, III, 1, 10); Diogenes Laertius's *The Lives and Opinions of Eminent Philosophers* (Hicks 1933 IX, 56). The three narratives of the story vary in some side details such as the amount of the payment agreed and etc., but the core idea of the paradox appears similar in all three of them. Cf. Ахведиани 2010, where a different account of how the payment has been agreed is given, which corresponds to none of the three sources. A similar story is told about the dispute between Corax, a 5th c. B.C. Sicilian rhetor, and his pupil Thysias (Cole 1991).

and (ii) play in the argumentation as presented in the original story, and the major confusion of the actions to which (ii) refers, with the different linguistic descriptions of them. Our actional paradox solution clarifies the major confusion and with this help resolves the minor one, too.

Let us consider (i) and (ii) in more detail. It is easy to see that challenge (ii) can be reformulated as

(ii') how Euathlus can legitimately avoid paying the tuition fee to Protagoras? Although conceptually (ii) and (ii') are presented as incompatible in the story, and once (ii) is satisfied and Protagoras gets paid by Euathlus, this is exactly the case when (ii') is not satisfied, which amounts to Euathlus not having escaped from paying the fee, so both (ii) and (ii') can play the role of a sound reason for a reply to (i) in an equally good manner.

Observe two aspects of this connection between (i) and (ii):

(a) Either of the replies to (ii) and (ii') can constitute the content of the reply to (i) and can be equally well justified for that by the arguments;

(b) When taken by themselves abruptly out of the paradox narrative, the two positive propositions describing the definite replies to (ii) and (ii') as well as their negative counterparts can be both satisfied and this makes no contradiction inside the two pairs, for the two linguistic descriptions of the situation in which (ii) and (ii') are both satisfied are perfectly compatible with each other, and this is so for the pair of their negative counterparts, too:

(ii-s) Protagoras gets paid for his instruction of Euathlus.

(ii'-s) Euathlus legitimately avoids paying the tuition fee to Protagoras.

(- ii-s) Protagoras does not get paid for his instruction of Euathlus.

(- ii'-s) Euathlus does not legitimately avoid paying the tuition fee to Protagoras.

What in fact makes (ii-s) and (ii'-s) appear as confronting each other is the empirical aspect of the situation according to which it is Euathlus who is expected to pay the fee to Protagoras but who would like to legitimately escape from doing so. Those Euathlus's actions that cannot be performed together at the same time by him are paying the fee and not paying it at the same moment, for whenever Euathlus pays the fee this immediately results in not having avoided it, and vice versa. However, the linguistic expressions employed to describe these two incompatible actions of Euathlus's paying and not paying may vary, and whereas some pairs of sentences used for that are clearly contradictory as *Euathlus pays the fee* and *Euathlus does not pay the fee*, or (ii'-s) and (- ii'-s), others are not so. An example of the latter is (-ii-s) and (ii'-s). This observation suggests the first hint of the paradox solution: the actions that are incompatible to be performed together

linguistically can be described by different pairs of sentences, some of which can be contradictories, while others are not so.

Let us develop this distinction between the actions and the propositions a bit further. The narrative of the paradox explicitly tells us of the two actions at question (along with their negative counterparts), namely, Protagoras's getting paid and Euathlus's paying, and these two actions, passive and active, apparently are meant to be performed by different agents: the former – passively by Protagoras, and the latter – actively by Euathlus. Despite this and the fact that they can be expressed by a pair of perfectly compatible propositions, the actions referred to in the positive replies to (ii) and (ii') cannot be realized together at the same moment inasmuch as it happens to the actions described by (ii-s) and (ii'-s): performing the action described by a positive reply to (ii) amounts to not performing the action which a positive reply to (ii') is taken to signify, and vice versa. Therefore, regarding the paradox's descriptive formulation we can use either of the replies' formulation (ii) and (ii'), positive or negative, and thus disregard the distinction between them, since it is a contingent and empirical distinction referring to the actions' propositional descriptions and not to the actions themselves. As concerns the two actions themselves, which play a crucial role in the paradox solution, we will carefully distinguish between them along with doing so with respect to their propositional description in (ii-s) and (ii'-s).

The fact that one of the paradox issues can be represented by two pairs of linguistically distinct propositions, which describe the incompatible actions although themselves are perfectly compatible to each other propositions, is the key aspect of our solution based on drawing the borderline between actions and propositions. Since the propositions describing the outcomes of the lawsuit do not contradict each other, unless they are taken not as the descriptions but as representations of the actions performed by the two disputants, we can conclude that the collision lies in the realm of actions and not in that of their propositional descriptions. In other words, although the conflict in the story amounts to the incompatible actions meant to be performed in reply to (i) and (ii) by some agents involved in the dispute, and not to the propositions used to describe the situations achieved as the result of what has been done by the agents, what makes the story look paradoxical is using two different pairs of propositions for describing one and the same pair of (incompatible) actions. The paradox arises due to the fact that the four propositions which form up these two pairs, namely (ii-s), (ii'-s), (– ii-s), (–ii'-s), can be paired in two distinct ways, one of which results in a contradiction, when a pair at question is made up by the proposition and its negation, for example (ii-s) and (– ii-s), or (ii'-s) and (–ii'-s), whereas the

other way gives a consistent pair of propositions, for example, (ii-s) and (ii'-s), or (– ii-s) and (–ii'-s).

In order to show this in section 2 we consider distinctions between the actions and the propositions and observe how these affect the practical arguments in which actions can play the roles of premises or conclusions. With the help of these distinctions in section 3 we analyze the paradox and in section 4 outline its legal and logical solutions suggested so far. In section 5 we propose our actional solution based on these two distinctions.

Section 2. Actions, propositions and practical arguments

The logical difference between actions and propositions amounts to two key properties connected to each other: we call them expressive and propositional. The propositional property concerns their abilities to be evaluated in terms of truth and falsity and, consequently, their capability to be premises or conclusions of the valid inferences. The expressive property shows how actions and propositions are handled with respect to how they are used to be signified. These two properties illuminate both what actions and propositions have in common as well as in what they essentially differ. Therefore, considering them will unveil those aspects of confusions between the actions and the propositions which stem out of when what differentiates them is taken to be equally common to them, and vice versa. We argue that two of such confusions are the sources of the paradox.

The expressive property can be viewed as a common property of actions and propositions, for they both can be expressed linguistically by means of certain sentences formulated in some language, and this common property may give rise to the confusion which facilitates the paradox. To find the tools for conveying actions other than using sentences of some language is far from being an easy task. One and the same linguistic expression can be used and is in fact often used to refer to the proposition as a mental entity expressing the action or as the factual result of performing it as well as the action itself. This is what we have in (ii-s) and (ii'-s) (and in their negative counterparts), each of which can be equally treated both as referring to the sentence in English expressing the proposition and to the action. It is the reason why many analysts view the paradox in court as a kind of a logical paradox in which some proposition ambiguously proves to be true along with its negation, as with *Euathlus pays the fee* and *Euathlus does not pay the fee*. Contrary to this, we propose to treat this paradox as a puzzle about colliding actions between which no such truth-value-based inferences are possible.

In the logical enquiry, irrespective of how rigor or vague the linguistic expressions used for reference to something might be, propositions and actions ought to be carefully discriminated. Propositions are mental entities employed to describe

some facts, actions, evaluations or any other objects to which the intellectual activity turns. Propositions can be true or false and, for logical reasons, whenever a proposition is true, this also means that its negation cannot be true at the same time. Thus in an inconsistent pair of propositions the truth of one of its members follows from the falsity of the other. These two aspects of propositions, namely, their truth-functionality and the applicability of the principle of non-contradictoriness to them, enable them to be premises or conclusions of the valid inferences in which the truth value is transmitted from the true premises to the conclusions, thus turning the latter necessarily true.

Actions are not mental entities like propositions but are empirical facts. They neither describe anything, nor can they serve as premises or conclusions of inferences in the way propositions do. Actions cannot be evaluated as true or false in the same way as propositions. Moreover, contrary to what the non-contradictoriness principle says, one and the same action as long as it has not been performed yet can be both realized and left unrealized by the same agent. Thus, in the paradox, Euathlus has two options of how to act at the same time: he can pay the fee inasmuch as he can refrain from doing so. These two options do not differ in what concerns the actions themselves which, of course, are impossible to be performed together. The difference is rather empirical, with respect to the situation or to Euathlus's personal preferences what to do. After an option to act is realized by an agent, the other option ceases to be realizable, and that is why Euathlus's paying the fee according to the contract means his not-paying it according to the verdict. It is crucial to differentiate the empirically based relations inside the pair of the incompatible actions and the logical relations between the propositions in the inconsistent pair of them: the former are causal and they are relations between the empirical facts whereas the latter are mental and they are based on the truth values of the propositions.

Nevertheless, actions can play the roles of premises and conclusions in the practical arguments which contrary to the deductive arguments can be construed in terms of transmitting actions' satisfiability and not in terms of that of truth values.² Such are legal decisions including those giving a reply to (i). Beside being an action itself, a legal decision changes the status of a definite action. For example, the jurors' decision makes obligatory any of the two actions, Euathlus's paying or non-paying the fee. Such arguments are called practical in contrast to the propositional arguments, which may be called theoretical in this respect. The ar-

² We take the satisfiability in an actional sense, as pointing to the fact that the action is performed, and not in a descriptive sense, as a corresponding indicative proposition describing this fact. We abstract ourselves from the controversial question whether actions and descriptive propositions have the same propositional content or not.

guments providing replies to (ii) belong to practical ones, too. To say that actions are unable to serve as premises or conclusions of any valid arguments is just another way to maintain that the legal decisions which include actions as their necessary elements are either not the conclusions of some arguments (and that instead there exists something else that plays the role of such conclusions), or they are a signal of the arguments' invalidity. It is clear that practical arguments are not deductive and that their validity has to be determined otherwise than that of propositional inferences (Лисанюк 2015, 283–286), but this core difference in the arguments' validity definition does not prevent legal arguments from being valid arguments. The same can be said regarding the arguments expressing medical decisions, which are also practical (Лисанюк 2016).

Although the contradictory relations between a proposition and its negation cannot be plainly applied to actions, there are certain relations between actions that can do a similar job. These are incompatibility relations between actions. Before one of actions A or $\neg A$ is realized by an agent α both doing A as well as refraining from it - $\neg A$ are available to α . Contrary to this, after α has chosen A (and did A), $\neg A$ - doing becomes no longer available to α (in the same situation), and vice versa (for choosing $\neg A$). If the jurors in their reply to (i) decide that Euathlus has to pay the fee, this immediately rejects their possible decision to support Euathlus's claim. Such incompatibility of actions is practical and not truth-functional, for it depends on the empirical choice of the agent, and it must not be confused with the truth-functional relations between the proposition describing the resulting situation after the action has been performed with its negative counterpart.

The rational justification of such agentive choice, whenever it exists, is an argument that demonstrates why the agent has picked out this particular alternative of the action done. At the same time this argument also demonstrates why all other actional options have been rejected by the agent (Searle 2001, 265). In this respect the practical argument functions in a way similar to a valid propositional inference, the conclusion of which refutes the contradictory conclusion of the contrary inference. Therefore, since actions and propositions differ in the terms used for their evaluation and since they both can give rise to some cases of incompatibility, such cases will necessarily differ in the kinds of the incompatibility relations established between their legs and thus will require diverse approaches for their solution. The actional incompatibility should not be confused with the propositional inconsistency, in spite of the fact that these two are the kinds of incompatibility.

Section 3. The analysis of the Paradox

In the paradox there are three essential issues that generate ambiguities when Protagoras and Euathlus switch their positions regarding those issues in their practical arguments. They are

- (1) Does the contract include the cases in which Euathlus is a party of the conflict?
- (2) Does this lawsuit comply with the contract?
- (3) Does Euathlus have to pay the fee?

Let us call them Contract dispute, Lawsuit dispute and Fee dispute respectively. According to the paradox narrative, Protagoras and Euathlus have two lines in their argumentation, each of which is construed out of Protagoras's and Euathlus's positive or negative replies to (1) – (3), we will call them P- and E-lines:

P ₁		E ₁	
(1-)	No, the contract does not include the cases in which Euathlus is a party of the conflict	(1+)	Yes, the contract includes the cases in which Euathlus is a party of the conflict
(2-)	No, this lawsuit does not comply with the contract;	(2+)	Yes, this lawsuit complies with the contract
(3+)	Yes, Euathlus has to pay (according to the jurors' decision)	(3-)	No, Euathlus does not have to pay (according to the jurors' decision)
P ₂		E ₂	
(1+)	Yes, the contract includes the cases in which Euathlus is a party of the conflict	(1-)	No, the contract does not include the cases in which Euathlus is a party of the conflict
(2+)	Yes, this lawsuit complies with the contract	(2-)	No, this lawsuit does not comply with the contract
(3+)	Yes, Euathlus has to pay (according to the contract, as he has won)	(3-)	No, Euathlus does not have to pay (according to the contract, as he has lost the case)

It is easy to see that depending on the conclusions needed in support of their claims, both Protagoras and Euathlus in one argumentation line adopt a positive reply to one of the issues (1) – (3), but choose its negative counterpart in the other. Observe that although the replies are linguistically expressed by the pairs of the inconsistent propositions, in fact they are not propositions themselves but actions of adopting certain replies in the framework of a definite line. Due to being actions and not propositions they are compatible together at the same moment, as long as one of them is not satisfied. Besides, choosing a positive or a negative reply is equally available to Protagoras and Euathlus. Contrary to that, asserting the contradictory pairs of the linguistic sentences expressing these actions will inevitably

result in the inconsistency of the argument. Therefore, both Protagoras and Euathlus do not theoretically draw valid conclusions from the premises construed out of the propositions expressing their replies to (1) – (3), picking either positive or negative ones, but not altogether in one inference. Instead they perform actions of choosing between the options of the replies available to them for the sake of producing practical arguments in support of their (practical) claims. This suggests that what we face in the story is no logical paradox which would imply truth-functionally ambiguous propositions but a kind of game-like practical reasoning of weighing pros and contras.

Section 4. The paradox solutions

In this section we will show that majority of the paradox solutions, which fall into two groups, legal and logical, amount to drawing additional distinctions for the sake of making some arguments in E- and P- lines weightier than others.³ Doing so can help to find a reply to (i) by means of justifying preferences in reply to (ii), which otherwise is far from being easy to adopt as uncontroversial. We will also demonstrate how these additional distinctions pave their diverse ways for discriminating either among the propositions representing different aspects of the actions referred to in the paradox narrative, or between the actions and the propositions describing them.

The jurors who concluded to deliver no verdict for the case apparently had been seeking for a decision the evidences in support of which would have been at least a shred stronger than any of the objections that might have been cast against it, once no irrefutable decision is available in the case. However, since the jurors seem to have treated both decisions available – that Euathlus is to pay the fee and that he is not to pay it - as equal in terms of their ability to be justified and refuted, they tried to refrain from deciding the case. As E- and P- lines demonstrate, each of the two decisions has both a justification and a rejoinder to it, what results in equally good arguments in favor or against the two in the table thus making any of would-be taken jury's decisions refutable in their core issues.

Three aspects regarding this strange output are essential in our discussion of the paradox solutions. One of them has to do with the jurors' 'no verdict' for the case; the other one recapitulates distinctions between logical and legal paradox solutions; the third brings to light a conceptual property characterizing all the suggested solutions. As we will see, the first aspect, which we will demonstrate

³ We abstract from discussing the game-theoretical solutions in a separate group, since they develop the perspective of calculating the disputants' contributions and arguments in the vein of adding extra scores into their positions by means of their refinements. Cf., for example, Светлов 2005, 372.

below, follows from the second, and it is the first one that will give us a clue how to classify the solutions.

In fact, by 'no verdict' the jurors did not fully succeed in refraining from deciding the case, for although their mute step down may be viewed as accomplishing the goal of giving no reply to (i), there is a good reason to say that they did not avoid deciding the case. The jurors' 'no verdict' made the premise describing their (possible) definite decision invalid for both P- and E-lines of argumentation, which indirectly supported (E2) line. Their 'no verdict' step to some extent plays into Euathlus's hands, since it secures the situation just as it appears when Protagoras has been ineffectually waiting to be paid following Euathlus's success in court, and as neither of these did happen, he went to the court himself. The jurors' refusal to decide the case leaves Euathlus's debt to Protagoras unpaid, which is quite proximate to the objective Euathlus has been arguing for. For this reason, many analysts consider such jurors' conduct as legally or procedurally wrong,⁴ and point to the fact that it is the jurors' core obligation to decide the cases in front of them, and the only way to avoid executing this obligation for a juror is to step down.

Observe that the jurors' decision is not just a collection of sentences expressing some events' evaluations meant to be accepted by the parties of the conflict but instead is a performative act which implies definite consequences imposed on the parties who by means of the decision are made obligated to obey it in their future conduct. The jurors' 'no verdict' implicitly supports Euathlus's claim, for although it does not cancel his disputed debt, contrary to Protagoras's claim it imposes no obligation on Euathlus. In Fee dispute both E- and P-lines refer to the jurors' definite action as to their decision of the case, but their 'no verdict' corresponds to neither of the two possible actions expected in reply to (i). Consequently, at the moment when in favor of their claims Protagoras and Euathlus advance their practical arguments in which they incorporate the premises referring to the jurors' action of providing a decision of the case, no such jurors' action exists and all the options for making it are still equally open to be picked out by the jurors. This prompts to look for some additional temporal distinctions between the actions which would discriminate those realized and thus eligible for being decisive premises in the practical arguments from those not realized and ineligible for that so far. The juror's 'no verdict' falls into the latter group.

The so-called 'two-case' solution to the paradox proposed by many analysts including Leibniz, Valla, Smullyan is based on the idea of a temporal distinction and it purports to bring back the premise of the definite jurors' decision to the argument. This solution discriminates between (1-) and (1+) and (2-) and (2+) in

⁴ For example, L. Valla (Nauta, Copenhagen 2012, III 13, 3-31), Leibniz (Artosi et al. 2013, 88), R. Smullyan (Смаллиан 2005, 131).

such a way that in the earlier lawsuit the jurors reject Protagoras's claim because of (1-) and (2-) and thus adopt (E2), but in the later one they take the earlier verdict as the additional premise that validates their second decision in the vein of (P2). In this way, the idea of adding some temporal distinctions is inspired by logical reasons of finding a firm premise for the conclusion and contributes to the legal 'two case' solution (Lenzen 1977).

A substantial explanation for the 'no verdict' step of the jurors can be found in the very distinction between the legal and the logical solutions of the paradox. Regarding the legal 'two-case' solution, this distinction relies on a logical temporal discrimination which provides the necessary additional premise and thus makes one of the genuinely equally powerful (E2) and (P2) lines of argumentation weightier. Generally, the legal solutions aim to propose the ways of how to justify the jurors's reply to (i), whereas the logical solutions suggest how to create auxiliary and good premises for that by means of replying to (ii). To put it otherwise, the legal solutions show how to act in (i) and the logical ones provide persuasive arguments for that in (ii). This is one of the reasons why some recommendations in the legal group do not rely on any of the logical solutions, trying to stick to legal aspects either of the dispute itself or of the judicial ways of its resolution in the court (Sobel 1987, Jankowski 2015).

The legal solutions fall into four groups.⁵ The first is the two-case solution already mentioned above. Another group of the legal solutions called 'unduly' develops the idea of adopting (E2) line. According to the 'unduly' solutions, the jurors may look for auxiliary evidence. For example, they may investigate why Eauthlus did not start his career in court after completing Protagoras's course as he had been intending to; or they may observe the principle according to which any service including Protagoras's instruction to Eauthlus deserves a fair price to be paid for; or they may just point to the undue time for Protagoras's claim thus encouraging him to wait until Eauthlus starts his intended career along with the recommendation to Eauthlus to take on a case. The third group of legal decisions amounts to the idea of calculating the arguments with respect to the burden of proof in the dispute, and in this vein Protagoras loses his lawsuit. Since Protagoras's and Euathlus's arguments are equal in weight, the disputant bearing the burden of proof loses. As Protagoras is a plaintiff, it turns the score in favour of Euathlus. The fourth group of the legal solutions may be called a counter-historical one. In Ancient Athens, citizens, including both defendants and accusers, were obliged to represent themselves in court, but outside the courtroom it was allowed to hire a logographos for preparing a speech which then would be personally presented afore the jurors. But for this rule of self-representation, Eu-

⁵ See a discussion of these in Лисанюк, Микиртумов 2009 and Ивин 2004, 269–271.

athlus could hire a lawyer, thus taking this case out of the scope of his contract with Protagoras, making it invalid as a subject case for the payment and replacing premises (1) and (2) in all four lines with some newly developed argumentation.

The logical solutions, falling into two groups, suggest how to validly justify or refute some of the possible legal solutions. The first group observes additional distinctions in Protagoras's and Euathlus's argumentation and it includes the 'temporal' solution discussed above. L. Åquist proposes to rank the conditions mentioned in the contract into necessary and sufficient thus turning the latter into stronger arguments' premises than the former (Åquist 1995). For example, we can treat premise (1) as sufficient, which will make premise (2) necessary, or vice versa. This approach can assist in ruling out the problem of the equal strength of the justification and the refutation and in this way provides support for a certain legal decision. Åquist's solution clearly develops the perspective of discriminating among the propositions in the vein of the 'two-case' solution, inasmuch as all the rest logical solutions do. Another proposal in this group advances the idea to formulate the notions of 'the first case' and 'the first won case' in a more rigorous way, preferably with the help of Russell's theory of logical types (Northrop 1944, 51-52). Thus it contributes into choosing between the positive and the negative replies in Court and Lawsuit disputes. In some sense all the three proposals, 'temporal', 'necessary \ sufficient' and 'logical types' are designed in a similar way to the effect of clarifying the contract conditions by means of expanding them with some further distinctions. Furthermore, all the three match with either 'two case' or 'unduly' legal solutions, for they enlarge the number of arguments involved and thus break the initial equational balance between the justification and the refutation of the possibly replies to (i). However, all these logical proposals are ad hoc patches, for although they suggest some extra premises to the effect of a more detailed description of the paradox narrative, they leave the initial issue (ii) opt to the jurors' verdict, which due its being an action remains ineligible to serve as a premise before being realized. Such patches propositionally bypass the paradox resolution at the expense of its actional grace.

The same can be said as regards the second group of the logical solutions, treating the paradox as a particular case of self-reference and suggesting two negative accounts of it, which can be both viewed as an invitation for further inquiry into the paradox roots. According to one of them the paradox is just an incorrect application of a condition to itself: the condition in the contract refers to the jurors' decision, which refers back to the condition in the contract. The second negative account points to the invalidity of the arguments on both sides, for the contradiction in the arguments equally cancels them. The reason for this is that in the antecedents of the conditionals used as their arguments Protagoras

and Euathlus employ inconsistent pairs of propositions which consist of a proposition and its negation.

The third aspect of the jurors' 'no verdict' stems from the core perspective to which all the so far proposed legal and logical solutions belong, namely, from the fact that by hook or by crook they all end up with obligating Euathlus to pay the disputed fee to Protagoras. In the context of this perspective, only 'no verdict' takes a stand which can be viewed either as a mute support of Euathlus's claim, or as... giving birth to this centuries long debate about how to solve the paradox, the debate which again seems to bring grist to Protagoras's mill in the vein of the proposed solutions. In the next Section we will show why Protagoras is in fact the only beneficiary of this sophisticated story.

5. The actional solution

There are three aspects of our actional solution: actional proper, cognitive and historical. The cognitive and the historical aspects are derivative and complimentary, for they rely on the actional solution with which we will start considering them in turn.

E- and P-lines of argumentation give us four possible routes but they lead to just two clearly distinct outcomes, $E(3-)$ and $P(3+)$. This perfectly corresponds to our differentiation between actions and propositions, for there are only two actions at question - paying and not-paying – in favor or against which the argumentation lines are construed. Recall that the incompatibility of actions and the inconsistency of propositions are epistemologically different: the former stems from the empirical fact which has to do with certain physical agentive impossibility to perform the two (or more) definite actions at once, whereas the latter amounts to the logical impossibility for the proposition and its negation to be true at once where empirical impossibility plays a contingent role. In other words, whenever two actions are incompatible, they are so empirically and thus irrelevant to whether they are a pair of performing something and refraining from it, or just two actions which cannot be realized together by one agent. An inconsistent pair of propositions is always made up out of a proposition and its negation, and although such a pair of propositions may describe what in fact exists along with its non-existing counterpart, the inconsistency relation is established between the two propositions as mental entities and not between the two facts. With this in mind let us now look at how these two actions are described in E- and P-lines.

The action described as not-paying in $E_1(3-)$ – according to the jurors' decision – is the same action of paying as described in $P_2(3+)$, for although the two descriptions are propositionally inconsistent with each other, nevertheless, they

both symbolize one and the same action when Euathlus pays the fee to Protagoras after winning his case, which amounts to his not-paying according to the contract. Similarly, the action propositionally referred to as not-paying in $E_2(3^-)$ – according to the contract, after Euathlus has lost the case – is identical with the action referred to as paying $P_1(3^+)$ due to the jurors' decision. This is so due to the fact that in the paradox narrative the empirical incompatibility between Euathlus's paying and not-paying at the same time sheds light either towards the descriptively presented reasons for these actions to be performed or not, or in the direction of the jurors' whose actions ambiguously are employed as a certain premise. Most of the legal paradox solutions discussed above pursues a way to draw distinctions among these diverse descriptions of the actions.

In our everyday conversational discourse one and the same action often can be linguistically expressed by inconsistent propositions which may illuminate its aspects in an incompatible manner, and such examples are seldom treated as paradoxical. For example, the action of marrying Suzy Mae performed by John Dow can be propositionally described as both his marrying her for love, his marrying her for, say, career and not for love, as well as his marrying her because of their official engagement to each other. On the other hand, John's marrying Suzy Mae, for love or otherwise, is John's not-marrying any other girl at the same moment. Despite the fact that John's initial intention was to marry Suzy and not to not-marry anyone else, his action of not-marrying is rather regarded as the causal consequence of the former than its different linguistic description, inasmuch as in the inconsistent pair of propositions the negation of one of them is implied by the negation of the other.

Consequently, the actional solution of the paradox brings us to the contention that Euathlus pays the fee to Protagoras anyway, which amounts to both (ii-s) and ($- ii$ '-s) if we take the action of Protagoras's getting paid by the contract as the same to Euathlus's action of avoiding paying by the verdict. This can be described as both (ii'-s) and ($- ii$ -s), too, if we treat the action of Euathlus's not-paying by the contract as the one of Protagoras's getting paid by the verdict. The phantom contrary action descriptions of the paradox narrative represent these two actions of Euathlus and Protagoras, each along with its ghost alternative, and does so by means of the two pairs of the inconsistent propositions, (ii-s) vs ($- ii$ -s) and (ii'-s) vs ($- ii$ '-s), which ambiguously imply each other's negations and which follow from another pair of the propositions that are used to describe actionally incompatible decisions of the jurors, which are likewise both possible before a verdict that once given turns them incompatible. Keeping to the actions themselves and not to the propositions representing them in analyzing the narrative efficiently breaks this circuit of phantomlike descriptions of already realized and

would-be actions and unveils what is done indeed. Therefore, the actional solution we advocate here is perhaps best represented by concluding that Protagoras gets paid anyway, irrespective of how this action is described.

Let us now turn to the cognitive and historical aspects of our paradox solution which also provide support for both the contention that Protagoras gets paid anyway in this dispute and for choosing the terms of paying rather than not-paying for its representation. The cognitive aspect suggests an explanation for our conclusion that there are only two strategies in four E- and P-lines of argumentation, (1-), (2-) and (1+), (2+), and that the two discussants are identical in how they employ them. Although the two lines seem to imply four outcomes, in fact there are only two of them, as we have just observed, and even this pair of dispute outcomes describes one and the same action. Are there two distinct discussants then? If yes, how to discriminate between them in this dispute then? One may look for a cognitive difference in how they think or behave, but the paradox narrative gives no clue there. As presented in the narrative, neither the actions the two perform (or are intended to), nor their beliefs advanced in the arguments prompt where the cognitive borderline between Protagoras and Euathlus is. Consequently, it must be concluded that there is only one agent who is presenting his narrative of the sophism, and it is Protagoras. This is the cognitive reason why Protagoras gets paid anyway: just because it is his story of how he got paid anyway, be it by his pupil Euathlus or by anyone else. The historical aspect of the solution amounts to the fact that other than in the paradox narrative there is no historical evidence that Euathlus has ever existed, as far as I know.

6. Conclusion

There are three reasons why Protagoras gets paid anyway in this story. The actional solution relies on the two kinds of the distinctions, propositional and expressive, between sentences formulated in some language and used to describe Protagoras's and Euathlus's actions. The propositional distinction shows why Euathlus's paying the fee according to the contract is identical with his not-paying it according to the jurors' decision, and vice versa. In this way the propositional distinction provides the core actional solution of the paradox. The expressive distinction explains how and why one and the same action can be described by pairs of the inconsistent sentences, and sheds light on the confusions between the actions and the linguistic expressions employed to describe them. In this vein the expressive distinction illuminates the roots of the paradox.

In the paper, we have observed the paradox narrative as an example of practical argumentation and outlined the four lines in such argumentation, two for Protagoras and two for Euathlus, what has led us to the conclusion that in fact

there are only two lines of argumentation, not four of them, and they both arrive at two sentences, and not at four of them. Moreover, these two sentences describe one and the same action.

The cognitive and the historical aspects of our paradox solution suggest that Euathlus is a ‘phantom’ in this narrative, which is Protagoras’s story of how he gets paid anyway irrespective of who his disciple was. The cognitive aspect points to the fact that whenever the actions and the arguments of two agents in the dispute are identical no distinction can be drawn to discriminate between them. Therefore, there is just one agent, and it is Protagoras, for there is no historical evidence of Euathlus’s existence other than this sophism.

REFERENCES

This paper is an extended version of Lisanyuk 2015a. I am indebted to E. Dragalina-Chernaya, I. Mikirtumov, B. Guerts, A. Tamminga, V. Dolgorukov, V. Kulikov and D. Tiskin for their helpful comments.

- Åquist, L. (1995) “The Protagoras Case: An Exercise in Elementary Logic for Lawyers,” Bjarup, J., Blegvad, M., ed. *Time, law, and society : proceedings of a Nordic symposium held May 1994 at Sandbjerg Gods, Denmark*. Stuttgart: F. Steiner, 73–84.
- Artosi, A., Pieri, B., Sartor, G., ed. (2013) *Leibniz: Logico-philosophical puzzles in the law*. Springer.
- Butler, H. E. transl. (1921) *Quintilian, Institutio oratoria*. Vol. I. London: William Heinemann, NY: G.P. Putnam’s sons.
- Cole, T. (1991) “Who was Corax?” *Illinois Classical Studies* 16, 65–84.
- Hicks, R., ed. (1933) *Diogenes Laertius: Lives of Eminent Philosophers I & II*. Cambridge, Mass.
- Jankowski, B. (2015) “The rhetor’s dilemma: Leibniz’s Approach to an Ancient case,” Armgardt M., Canivez P., Chassagnard-Pinet S., ed. *Past and present interactions in legal reasoning and logic*. Springer, 95–107.
- Lenzen, W. (1977) “Protagoras contra Euathlus. Betrachtungen zu eine sogenannten Paradoxie,” *Ratio* 19, 164–168.
- Lisanyuk, E. (2015a) “Euathlus is virtual in the dispute about Protagoras’s tuition fee,” in E. G. Dragalina-Chernaya, ed. *Rationality in action: Intentions, Interpretations and Interactions*. St. Petersburg, 102–104.
- Nauta, L.; Copenhaver, B. P. (2012) *Lorenzo Valla. Dialectical Disputations*. Vol. 2. Cambridge, Mass: Harvard University Press.
- Northrop, E. P. (1944, repr. 1975) *Riddles in Mathematics*. Rogert E. Krieger, Huntington.
- Rolfe J. C., transl. (1927). *The Attic Nights of Aulus Gellius*. Vol. I. Cambridge, Mass.: Harvard University Press.
- Searle, J. (2001) *Rationality in action*. MIT Press, Cambridge, Mass.
- Sobel, J. H. (1987) “The law student and his teacher,” *Theoria* 53 (1), 1–18.

- Ахвледзиани, А. Н. (2010) “Гносеологический анализ возможных решений древнегреческого парадокса «тяжбы Протагора с Эватлом»,” *ΣΧΟΛΗ (Schole)* 4.2, 291–297.
- Ивин, А. А. (2004) *Логика для юристов*. Москва.
- Лисанюк, Е. Н. (2015) *Аргументация и убеждение*. Санкт-Петербург: Наука.
- Лисанюк, Е.Н. (2016) “Практическая аргументация и античная медицина,” *ΣΧΟΛΗ (Schole)* 10.1, 214–241.
- Лисанюк, Е. Н., Микиртумов, И. Б. (2009) “Дело «Протагор versus Эватл» принять нельзя отклонить,” Солонин, Ю. Н. и др., ред. *Человек познающий, человек создающий, человек верующий*. Санкт-Петербург: Изд-во СПбГУ, 85–97.
- Маковельский, А. О. (1967) *История логики*. Москва.
- Светлов, В. А. (2005) *Конфликт: Модели. Решения. Менеджмент*. Санкт-Петербург: Питер.
- Смалиан, Р. М. (2005) *Как же называется эта книга*. Москва.