

СТАТЬИ / ARTICLES

THE ARGUMENT ABOUT THE RIVER PROVIDED BY HERACLITUS OF EPHESUS AND THE NEED FOR A TEMPORAL DIMENSION IN ITS LOGICAL FORM

Miguel López-Astorga
Institute of Humanistic Studies “Juan Ignacio Molina”
University of Talca (Chile)
milopez@utalca.cl

ABSTRACT. The theories accounting for cognition based on formal schemata often claim that there is a logic in the human mind. From the thesis on the river given by Heraclitus of Ephesus, in this paper, it is argued that, if that logic exists, it cannot be simple, and that, at a minimum, it requires the assumption of some kind of temporal elements, which, in general, seem not to be considered in such theories. In particular, some reflections about possible ways those elements could be taken into account are presented.

KEYWORDS: Formal schemata, Heraclitus of Ephesus, logic, reasoning, time.

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Introduction

Currently, there are several formal theories supporting the thesis that there is really a logic in human thought, one of the most important approaches in this way being, perhaps, that of the mental logic theory, which proposes even a basic syntax in our mind (e.g., Braine & O’Brien 1998a; O’Brien & Li 2013). However, these theories still have great challenges to face. One of them is that they need to prove that, indeed, it is possible to capture all of the arguments and inferences that can be expressed in natural language by means of the formulae of a formal logical language, and this is exactly the problem that will be addressed in this paper.

Thus, my hypothesis here will be that, if it is true that a logical language of that type can be supposed, that language must have certain complexity and include, as a minimum, a temporal dimension, and, therefore, be able to differentiate moments in time. This point will be shown by means of a well-known argument offered by an ancient philosopher, Heraclitus of Ephesus. In particular, the argument is that related to the impossibility for anything to be immersed in the same river twice, and the intention will be to explain how, certainly, that is an argument that can only be linked to a logical form if we resort to formulae referring temporality.

But to do all of this, I will firstly present the fragment in which the argument is attributed to Heraclitus and try to contextualize it with some more data. Secondly, I will describe the general characteristics that the formal theories often have. And thirdly, I will try to show that Heraclitus argument raises difficulties that the usual machinery of those theories cannot easily remove, and that such difficulties can disappear only if such theories are supplemented with resources to capture scenarios changing in time. So, the next section is devoted to the argument provided by Heraclitus.

You cannot immerse yourself in the same river twice

The Heraclitus' argument that will be analyzed in this paper is to be found in the dialogue *Cratylus* (402 A) authored by Plato. It is as follows (see also, e.g., Fragment 215 in Kirk, Raven, & Schofield 1987):

λέγει που Ἡράκλειτος ὅτι πάντα χωρεῖ καὶ οὐδὲν μένει, καὶ ποταμοῦ ῥοῆ ἀπεικάζων τὰ ὄντα λέγει ὡς δις ἐς τὸν αὐτὸν ποταμὸν οὐκ ἂν ἐμβαίης.

[Heraclitus somewhere says that all the things are moving and nothing remains immobile, and comparing beings to a river current, he says that you could not immerse yourself in the same river twice].

In principle, the sense of this fragment seems to be obvious: everything changes in the same way as rivers, which never have the same water in different moments. In two distinct moments, the water of a same river is different, and this makes that very river different in those two moments. Thus, nothing is the same in two points in time.

Fragment 216 in Kirk et al. (1987), which corresponds to a text written by Aristotle (*Physics* Θ 3, 253b 9), clarifies further the idea of the argument:

καὶ φασὶ τινες κινεῖσθαι τῶν ὄντων οὐ τὰ μὲν τὰ δ' οὐ, ἀλλὰ λανθάνειν τοῦτο τὴν ἡμετέραν αἴσθησιν.

[Some people do not even state that some beings are moving and some beings are not, but that all of the beings are constantly moving, even though this fact is beyond our sensory perception].

Nevertheless, it cannot be said that this argument does not cause discussions between the specialists. For example, Kirk et al. (1987) and Guthrie (1962) give different interpretations of it. But the exact meaning of the argument when understood from the general framework of Heraclitus' philosophy is not the most interesting aspect of it in this paper. Here, it is enough to literally consider the example that is attributed to Heraclitus by Plato in *Cratylus*, that is, that it is not possible for anything to be in the same river twice, the reason of that being that, in the second moment, the river would be no longer exactly the same. Water passes and hence the specific water that is part of the river in a point in time cannot be part of that very river in another point in time. As shown below, this idea cannot be described by just simple well-formed formulae of classical first-order predicate logic. It is necessary to introduce, at least, elements referring to time. However, before explaining why this is so, the next section describes general theses habitual in the formal theories.

A logic for the human mind: the formal theories

As indicated, there is not a formal theory, but different theories that, without sharing exactly the same theses, agree that the human mind works by applying formal schemata to the logical forms that correspond to the sentences in natural language. In this way, clearly, a proposal can be (and, in fact, has been) to think that human thinking strictly follows standard logic and hence to link reasoning to papers such as those of Gentzen (1934, 1935). Nonetheless, many experimental results reported in the literature on cognitive science that cannot be accounted for by supposing that our mind just considers rules such as the ones of classical logic to make inferences have led to more elaborated versions of formal theories.

Indeed, results such as those mainly obtained with the four-cards selection task (e.g., Wason 1966, 1968) and arguments such as those summarized in papers such as the one of Johnson-Laird, Khemlani, and Goodwin (2015) showed very soon that it is not very probable that human beings make inferences simply using the formal rules of classical logic, since, if this were so, the majority responses in several reasoning tasks should be different from the usual ones. In this way, various logical approaches intending to explain the differences between the responses predicted by classical logic and those that are actually got (that is, to explain the errors from the classical logic point of view) have been presented, including

that of Henlé (1962) or that of Rips (1994). However, maybe, the strongest framework in this direction is nowadays the one of the mental logic theory, which has already been mentioned.

Certainly, the mental logic theory accounts for facts such as those indicated by insisting that the mental logic in our mind is not classical logic, even though they are very similar (e.g., O'Brien 2014). Perhaps, it might take too long to comment on all of the aspects of the mental logic theory that differentiate it from classical logic and, therefore, in its view, enable to understand the mistakes in reasoning habitually made by people. Nevertheless, some of the most well known of them can be enough to illustrate what this theory is exactly. Firstly, it proposes that the fact that a particular schema is valid in classical logic does not necessarily imply that that very schema is part of the logic of thought. Empirical evidence shows which of those schemata are really generally applied by individuals and, based on that information, the proponents of the theory clearly point out the rules that are natural in the human mind, which are those that people more often use, and the specific circumstances in which we can expect that such rules are used (e.g., Braine & O'Brien 1998b).

On the other hand, the adherents of the mental logic theory are also aware that individuals do not deal with contradictions in the same way as classical logic. This last logic allows deriving any well-formed formula in its system from a contradiction. And this is so by virtue of a well known principle: *Ex Contradictione Quodlibet Sequitur*. True, given two formulae such as α and $\neg\alpha$ (where ' \neg ' stands for negation), any formula β can be inferred in classical logic, the content of β being any possible content of any well-formed formula. Nevertheless, in mental logic inferences of this kind are not valid. Inconsistencies only reveal that some of the logical forms used in our inferential process is/are false. Therefore, contradictions only lead to review the previous mental contents and assumptions (e.g., Braine & O'Brien 1998c).

Furthermore, another very important point of the theory is that it acknowledges the role that general information had by people plays. Thus, when individuals make deductions, they do not only take the sentences explicitly present in the inferences into account, but also sentences, with their logical forms, expressing data known by them and which are in their long-term memory. These sentences are often called 'pragmatic premises' and, based on the mental logic theory, is very hard to understand the real human inferential activity without them (e.g., Braine & O'Brien 1998a, 1998c; Gouveia Roazzi, Moutinho, Bompastor Borges Dias, & O'Brien 2002).

There is no doubt that theses such as these ones give powerful resources to explain the actual human intellectual behavior. However, if we think about the

argument offered by Heraclitus indicated in the previous section, they can appear to be insufficient. As held in the next pages, that argument cannot be formally captured without some type of formal elements related to time. And, as far as I know, not only the mental logic theory lacks elements of this kind, but also the other formal theories.

The need for a temporal dimension

To show why a temporal dimension is necessary to capture Heraclitus' argument, I will resort to a language akin to the one of first-order predicate logic. As said, the logics proposed by the formal theories do not always absolutely match that logic (for the particular case of the mental logic theory, see, e.g., Braine 1998). Nonetheless, I think that this is not a problem for, at least, two reasons. On the one hand, I have indicated that I will only assume a language similar to that of first-order predicate logic, and not all of its rules and requirements, and this is so because I intend to make no derivation correct in first-order predicate logic and unacceptable in any formal theory. On the other hand, if one reviews the languages of the formal theories, it can easily be noted that translating expressions or formulae in the first-order predicate logic language into such languages is not difficult at all. So, it is evident that the explanation that I will present here can also apply to all of the formal theories.

Having said that, I just remind, before continuing, the meanings of certain symbols habitual in first-order predicate logic I will use:

\forall : Universal quantifier.

P: Predicate letter that, in this case, will refer to the action to be immersed.

x: Independent variable.

a, b, c: Constants.

\rightarrow : Conditional relationship.

\neg : Negation (it has already been defined).

In this way, given that the fragment in *Cratylus* mentions the action of entering (the verb $\epsilon\mu\beta\alpha\acute{\iota}\nu\omega$ is used) in the same river ($\epsilon\acute{\iota}\varsigma\ \tau\acute{\omicron}\nu\ \alpha\upsilon\tau\acute{\omicron}\nu\ \pi\omicron\tau\alpha\mu\acute{\omicron}\nu$), it is clear that it

is necessary for the predicate letter 'P' to be dyadic and enable the construction of formulae such as this one:

[I]: Pab (where 'a' represents, e.g., a particular person, 'b' stands for a particular river, and hence the entire formula means that 'a is immersed in b').

However, what the argument provides is that it is not possible to be immersed in the same river twice ($\delta\acute{\iota}\varsigma$), and, given the resources of a language such as the one of first-order predicate logic, maybe the best way to indicate that can be the following:

[II]: $Pab \rightarrow \neg Pab$

But [II] continues to be unsuitable for two causes. Firstly, it refers to only one individual, a, and the fragment seems to imply that nobody can be in the water of the same river twice. Secondly, what it really states is not that a cannot be immersed in b twice, but only that, if a is immersed in b (Pab), then a is not immersed in b ($\neg Pab$).

The first problem is not difficult to solve. It is enough to resort to the universal quantifier and transform [II] as follows:

[III]: $\forall x (Pxb \rightarrow \neg Pxb)$

Now, the formula expresses that, 'for all x, if x is immersed in the river b, then x is not immersed in the river b'. Nevertheless, it is clear that the second problem cannot be eliminated with resources such as those of classical first-order predicate logic alone. There is no doubt that the Greek word $\delta\acute{\iota}\varsigma$ refers to two different moments in time, and that circumstance can only be captured by means of temporal elements such as, for example, those corresponding to the semantics adopted by López-Astorga (2014). As in this last paper, without assuming all of the elements of temporal logics, we can take some aspects of them to deal with a particular difficulty. In our case, it appears that we only need a semantics of moments in time, that is, just one of the temporal elements adopted by López-Astorga (2014). That semantics can be described as a set $T = \{t_1, t_2, \dots, t_n\}$ of moments in time with characteristics such as those that, based on Vázquez (2001), are assigned to it in López-Astorga's paper, i.e., characteristics such as irreflexivity, transitivity, onward linearity, backward linearity, endlessness in the future, endlessness in the past, and density (López-Astorga 2014: 63; Vázquez 2001: 189). Thus, [III] could be transformed into:

[IV]: $\forall x [Pxb(t_i) \rightarrow \neg Pxb(t_j)]$ (where $i \geq 1 \leq n, j \geq 1 \leq n$, and $i \neq j$).

Obviously, [IV] is the formula that, amongst the four ones proposed until now, best expresses Heraclitus' idea, as it states that, 'for all x, if x is immersed in b at time i, x is not immersed in b at time j. As explained between brackets, both i and j are moments in time greater or equal to (\geq) 1, less than or equal to (\leq) n, and different from (\neq) each other. So, [IV] clearly implies that $\delta\iota\varsigma \ \acute{\epsilon}\varsigma \ \tau\acute{\omicron}\nu \ \alpha\upsilon\tau\acute{\omicron}\nu \ \pi\omicron\tau\alpha\mu\acute{\omicron}\nu \ \omicron\upsilon\chi \ \acute{\alpha}\nu \ \acute{\epsilon}\mu\beta\alpha\iota\eta\varsigma$, i.e., that you cannot be immersed in the same river twice, since, if you are immersed in it at a time such as i, you are not at a time such as j.

Of course, one might think that [IV] can be improved to better represent the argument, as, for example, because Heraclitus seems to refer to any river, 'b' may not be a constant and may be transformed in a variable akin to x and universally quantified. However, while this is actually so, it is also true that [IV] is illustrative enough to explain the point that this paper is intended to show, that is, that temporal elements are absolutely necessary in this case.

Nonetheless, a stronger possible objection against this account could be that temporal elements are not actually clearly needed, and that simple modal operators can be sufficient. Thus, it could be argued, for instance, that, without assuming a system such as K, which, as it is well known, is called in this way in recognition of the developments given by Kripke (e.g., Kripke 1959, 1962, 1963a, 1963b, 1965), in entirety, it would also be possible to resort to a modal language including, in addition to all the elements of classical first-order predicate logic, the modal operators of possibility (\diamond) and necessity (\square), the final result being a formula such as this one:

[V] $\forall x (Pxb \rightarrow \neg \diamond Pxb)$

But this formula has inconveniences too. On the one hand, what it expresses is that 'for all x, if x is immersed in the river b, then it is not possible for x to be immersed in the river b'. Evidently, apart from the fact that the sentence itself appears to make no sense (since it says that something happening is not possible), it does not describe what Heraclitus really wanted to claim, and this is clearly so because it lacks temporal dimension. On the other hand, following the usual definitions in the modal logic language, a formula such as $\neg \diamond \alpha$ is equivalent to $\square \neg \alpha$, which in turn means that [V] is equivalent to:

[VI]: $\forall x (Pxb \rightarrow \square \neg Pxb)$

And, expressed in natural language, [VI] is even more contradictory than [V], as it provides that ‘for all x , if x is immersed in the river b , then, necessarily, x is not immersed in the river b ’. Furthermore, although all of the elements of a system such as K are not assumed, it is obvious that the acceptance of \diamond and \square leads, by virtue of common sense, to the acceptance of some axioms as well. That is the case, for example, of the axiom named ‘Axiom of Possibility’ by Hughes and Cresswell (1968), which has a structure similar to the following:

[VII]: $\alpha \rightarrow \diamond\alpha$ (see also, e.g., Fitting & Mendelsohn 1998, p. 5).

Undoubtedly, [VII] requires that the antecedent of [V] implies the contrary of what it implies, i.e., that it implies $\diamond Pxb$, and not $\neg\diamond Pxb$. Accordingly, [VII] seems to show that [V] is, if not absolutely incorrect, at a minimum, unsuitable. So, only a modal formula such as the following could be acceptable with some certainty:

[VIII]: $\forall x (Pxb \rightarrow \neg\diamond Pxc)$ (where ‘ b ’ is at time t_i and ‘ c ’ is ‘ b ’ at time t_j , t_i and t_j keeping the characteristics attributed to them in [IV]).

But, again, apart from the fact that, as b in [IV], b and c could be quantified in [VIII] and hence this last formula could be much more complex, the explanation between brackets reveals that a temporal dimension of any kind is always necessary for Heraclitus’ argument. Even if we use some elements coming from a system such as K , such elements have to allow distinguishing moments in time to understand what the argument truly means. Therefore, it seems that, if we wish a logic that is able to describe the formal structure of the real arguments and inferences made by people, that logic must somehow include a temporal dimension.

Conclusions

It is not new that the philosophical thought in general and the Pre-Socratic thought in particular often cause problems to classical logic, since it is sometimes hard, from this last logic, to find the correct logical forms for the sentences and theses provided by thinkers (see, e.g., for the particular case of Thales of Miletus, López-Astorga 2017). However, this does not mean that there cannot be a logic with the potential to capture such sentences and theses, or that a mental logic is impossible. The only point that it seems to prove is that those logics cannot be as simple as the classical one.

Perhaps this fact also demonstrates that such logics cannot be even as basic as the systems presented by the formal theories for now, since, for example, as men-

tioned, it appears that none of those theories include clear elements to refer to temporal dimensions. Nevertheless, as it can be inferred from what has been shown in this paper, circumstances such as this last one do not prevent from speaking about formal schemata in the human mind either. In fact, what the previous pages really evince is that even arguments such as the one of Heraclitus of Ephesus commented on here, which, at least *prima facie*, appears to be intended to argue that reality is intrinsically contradictory and inconsistent (see, e.g., Kirk et al. 1987, or Guthrie 1962, for a discussion to check to what extent this is exactly so), can be expressed by means of a logical language. The only aspect that needs to be clarified is which language is the best one to carry out activities of that kind.

Accordingly, we cannot reject the hypothesis that a logic of thought exists yet. Maybe that logic is not, as said, that of the mental logic theory or any of those proposed by the other formal theories, but it is obvious that a logic of that type is absolutely possible. In this way, this paper makes it clear that, in addition to empirical and experimental studies, attempts to find the deep formal structure of the theses offered by some philosophers, especially if such theses try to provide very complex and abstract ideas, can also be useful to identify what that logic can be exactly.

Thus, one of the most important findings of these pages is that the argument about the river given by Heraclitus is actually very difficult to formalize without temporal elements. So, it would be interesting to review, with a methodology similar to that used here, other arguments presented by other ancient and modern thinkers in order to confirm whether or not such elements are absolutely necessary in a possible logic of thought, or to see whether or not they reveal that other modal logics elements are required as well. In any case, it is evident from the account in the previous section that the work in this direction can be a relevant line of research to explore.

Furthermore, it is also clear that it is absolutely true neither that logical forms cannot capture all the richness of what can be involved in natural language nor that that can only be done, as held by, for example, Johnson-Laird (2010), by means of iconic scenarios describing reality akin to those raised by Peirce (1931-1958). It has also been claimed that even iconic scenarios such as those ones can be expressed using logical forms (e.g., López-Astorga 2015). Hence, there is no doubt that the search of more or less systematic mechanisms to obtain logical forms should continue. It is important not only in fields of study such as information processing or philosophy, but also in areas such as linguistics and translation. And the relevance that modal language in general and a temporal semantics in particular can have in this activity is one of the points that this paper has highlighted.

REFERENCES

- Braine, M. D. S. (1998) "Steps toward a mental-predicate logic," in M. D. S. Braine & D. P. O'Brien, eds. *Mental Logic*. Mahwah, N. J., 273-331.
- Braine, M. D. S. & O'Brien, D. P. (1998a) "How to investigate mental logic and the syntax of thought," in M. D. S. Braine & D. P. O'Brien, eds. *Mental Logic*. Mahwah, N. J., 45-61.
- Braine, M. D. S. & O'Brien, D. P. (1998b) "The theory of mental-propositional logic: Description and illustration", in M. D. S. Braine & D. P. O'Brien, eds. *Mental Logic*. Mahwah, N. J., 79-89.
- Braine, M. D. S. & O'Brien, D. P. (1998) "A theory of if: A lexical entry, reasoning program, and pragmatic principles," in M. D. S. Braine & D. P. O'Brien, eds. *Mental Logic*. Mahwah, N. J., 199-244
- Fitting, M. & Mendelsohn, R. L. (1998) *First-Order Modal Logic*. Dordrecht.
- Gentzen, G. (1934) "Untersuchungen über das logische Schließen I," *Mathematische Zeitschrift* 39(2), 176-210.
- Gentzen, G. (1935) "Untersuchungen über das logische Schließen II," *Mathematische Zeitschrift* 39(3), 405-431.
- Gouveia, E. L., Roazzi, A., Moutinho, K., Bompastor Borges Dias, M. G., & O'Brien, D. P. (2002) "Raciocínio condicional: influências pragmáticas," *Estudos de Psicologia* 7(2), 217-225.
- Guthrie, W. K. C. (1962) *A History of Greek Philosophy, Volume I*. Cambridge.
- Henlé, M. (1962) "On the relation between logic and thinking," *Psychological Review* 69, 366-378.
- Hughes, G. E. & Cresswell, M. J. (1968) *An Introduction to Modal Logic*. London.
- Johnson-Laird, P. N. (2010) "Against logical form," *Psychologica Belgica* 5(3/4), 193-221.
- Johnson-Laird, P. N., Khemlani, S., & Goodwin, G. P. (2015) "Logic, probability, and human reasoning," *Trends in Cognitive Sciences* 19(4), 201-214.
- Kirk, G. S., Raven, J. E., & Schofield, M. (1987) *Los filósofos presocráticos: Historia crítica con selección de textos*. Madrid.
- Kripke, S. A. (1959) "A completeness theorem in modal logic," *Journal of Symbolic logic* 24(1), 1-14.
- Kripke, S. A. (1962) "The undecidability of monadic modal quantification theory," *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 8, 113-116.
- Kripke, S. A. (1963a) "Semantical considerations in modal logic," *Acta Philosophica Fennica* 16, 83-94.
- Kripke, S. A. (1963b) "Semantical analysis of modal logic I: Normal modal propositional calculi," *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 9, 67-96.
- Kripke, S. A. (1965) "Semantical analysis of modal logic II: Non-normal modal propositional calculi," in J. W. Addison, L. Henkin, & A. Tarski, eds. *The Theory of Models: Proceedings of the 1963 International Symposium at Berkeley*. Amsterdam, 206-220.
- López-Astorga, M. (2014) "Análisis de la tarea de selección por medio de una semántica temporal," *Praxis Filosófica* 38, 53-70.

- López-Astorga, M. (2015) "The disjunction introduction rule: Syntactic and semantic considerations," *Pragmalingüística* 23, 141-149.
- López-Astorga, M. (2017) "Thales of Miletus and the semantic possibilities of his view of the soul," *Aisthema* IV(1), 101-112.
- O'Brien, D. P. (2014) "Conditionals and disjunctions in mental-logic theory: A response to Liu and Chou (2012) and to López-Astorga (2013)," *Universum* 29(2), 221-235.
- O'Brien, D. P. & Li, S. (2013) "Mental logic theory: A paradigmatic case of empirical research on the language of thought and inferential role semantics," *Journal of Foreign Languages* 36(6), 27-41.
- Peirce, C. S. (1931-1958) *Collected papers of Charles Sanders Peirce*. C. Hartshorne, P. Weiss, & A. Burks, eds. Cambridge, MA.
- Rips, L. J. (1994) *The Psychology of Proof: Deductive Reasoning in Human Thinking*. Cambridge, MA.
- Vázquez, M. (2001) "Breve introducción a la lógica temporal," *Revista Laguna* 9, 187-198.
- Wason, P. C. (1966) "Reasoning," in B. Foss, comp. *New Horizons in Psychology*. Harmondsworth (Middlesex): 135-151.
- Wason, P. C. (1968) "Reasoning about a rule," *Quarterly Journal of Experimental Psychology* 20, 273-281.